# THE ELEMENTS OF ASTRONOMY

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# THE
# ELEMENTS OF ASTRONOMY

BY

D. N. MALLIK, B.A., Sc.D., F.R.S.E.
LATE SCHOLAR, PETERHOUSE, CAMBRIDGE,
PROFESSOR, PRESIDENCY COLLEGE, CALCUTTA,
FELLOW OF THE CALCUTTA UNIVERSITY

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## PREFACE

In the following pages, it has been my object to give a brief and clear account of those portions of Astronomy which can be dealt with, with the help of elementary Mathematics. In the selection of subject-matter and its treatment, I have been guided by my experience as a teacher of Astronomy, so that the book will, I trust, be found to meet the requirements of students by stimulating thought and ensuring scientific accuracy.

*August, 1921.* D. N. M.

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# HISTORICAL INTRODUCTION

1. The Science of Astronomy, *literally*, the Science which deals with the laws of stars or heavenly bodies, generally, is the most ancient of the Sciences—with its beginnings almost at the dawn of human consciousness. This indeed, would be *a priori* evident, for from the earliest ages, the beauty and the grandeur of sun-rise and sun-set and of the panorama that the night presents,—even to the least observant on-looker—must have excited men's wonder and a spirit of enquiry.[1] The hymns of the Rigveda to the sun, the dawn and the sky, probably represent the earliest articulate attempt of the human race to recognise the imperative workings of law in the phenomena of the heavens,—as yet but dimly perceived. But while the supreme mystery behind these phenomena remained and still remains unsolved, they presented a regularity, a rhythm which could not fail to be perceived and carefully observed even by the earliest man. Moreover, since Science deals with measurement, Astronomical phenomena came almost automatically within its domain. One was almost implicitly impelled to follow the path, traced on the sky, by the sun, the moon and the stars, from day to day, one was perforce led to note the position of the horizon, where the earth and the sky appeared to

[1] Cf. the Hindu Astronomical work, *Suryya Siddhanta*, Ch. XII, 2-8.

meet and the points in it,—by a reference to terrestrial objects,—where the heavenly bodies appeared and disappeared and appeared again in their sojourn, according to almost an immutable law,—if supremely, most impressively inscrutable, as it is even now, in the main. And while, the striking regularity of these phenomena compelled attention, they were so radically bound up with the ordinary experience of man's daily life, that some kind of measurement became almost a necessary part of the very routine of existence, from the earliest ages. Thus, according to Baily, accurate Astronomical observations had been made in India, probably before three thousand B.C., a conclusion which, as we shall presently see is justified on independent evidence. It is, moreover, conceivable that the sacrificial rites described in the Vedas were themselves astronomic in their origin.[1] In any case, as they were regulated by the position of the moon with reference to the stars, they must be held to presuppose accurate Astronomical observations, which had, thus, come to be a religious necessity, so that it is reasonable to argue, *a priori*, that an extensive Astronomical knowledge obtained in India, even in the Vedic times.

2. But, even in the most primitive nomadic stage, Astronomical measurements, especially that of time in terms of the solar day must have begun,—necessarily, at first, in a very crude form,—though thousands of years must have elapsed before it came to be recognised that the year was the natural unit and thousands of years, longer, before even approximate data were available for taking it to consist of $365\frac{1}{4}$ days; yet it is asserted in the book called Chuking (2205 B. C.) that the year was

[1] According to one writer *Indra* is essentially the personification of the summer solstice and *Vritra*, that of the constellation of Hydra, Indra's conquest of Vritra representing the arrival of solstitial rains.

known to consist of 365¼ days in the time of Yao (2357 B. C.), while the Indian Astronomers, according to some,[1] seem to have arrived at that conclusion—at any rate, at an approximate result—even before that date.

3. This was indeed a wonderful achievement. It was in the first place necessary to become assured that the same sun and the same moon were being observed from day to day and then to form an idea, as to the true shape of the celestial vault. At first, this would be taken to be a hemisphere, surrounding a flat earth, limited by the observer's horizon. Gradually, the knowledge that the celestial vault is a complete sphere would emerge and we have Chinese records nearly 5000 years old to indicate, not merely that they had already grasp of these facts but that they had learnt to describe the motion of the sun, by its change of position among the stars. It is conceivable, that the ancient Astronomers had learnt to recognise long before even this remote epoch, the distinction, between stars (or bodies whose position relative to each other remained unchanged and which pursued, always, practically the same steady circular path in the heavens) and planets or wandering stars (including the sun and the moon), which moved *among* these stars.[2] But this had to be preceded by the knowledge of the fact, so difficult to realise, that stars were shining even at midday, although hidden from view on account of the sun's rays. When, therefore, we read that Yao gave instructions to his Astronomers to determine the positions of the solstices and equinoxes and they determined these with reference to

[1] There is considerable difference of opinion on this and similar points. *The personal equation* of the writers on a subject like this, which intrinsically lends itself to speculation vitiates many an investigation, otherwise ingenious.

[2] This is clearly set forth in *Suryya Siddhanta*.

the stars which occupied those points of space, we realise that observational and theoretical Astronomy had reached a high stage of development in China, already, in 2300 B. C. It is, therefore, by no means improbable that such knowledge prevailed in India in the Vedic times and therefore an Astronomic interpretation of the Vedic hymns which, as we have already noted, has been attempted by some authors and which postulates a knowledge of solstitial and equinoctial points on the part of the Vedic writers would not, to that extent, be altogether fanciful.

4. Going back to Chinese records, we are told, further, that in 2159 B. C., the Royal Astronomers Hi and Ho failed to predict an eclipse and were accordingly executed, so that apparently, an eclipse was then regarded, as it is sometimes regarded even now, as an event of serious portent. This, indeed, is, by no means, strange. For if celestial phenomena, in general, excited wonder and a spirit of worship, an eclipse could not fail to be associated with a temporary cataclysm and would as such, be naturally regarded with a mysterious awe. And even as the motions of the sun and the moon produced obvious terrestrial phenomena—day and night and changes in the seasons (although it was long before their intimate relation was realised), so, there was nothing to indicate that these motions and the positions of stars as well as such naturally striking phenomena as the eclipses and the appearance of comets were not capable of exerting occult and even baneful influence on man. It was no wonder, therefore, that those who studied these phenomena were popularly credited with and gradually came to claim supernatural wisdom. Accordingly, Astronomy and Astrology came to be inextricably mixed up, in the infancy of the Astronomical Science.

5, Of more scientific interest is the evidence that

the above account affords of the knowledge that the Chinese appear even then to have possessed some rule for predicting the eclipses. This is hardly to be wondered at: *Rigveda Sanghita* gives a correct explanation of the phases of the moon or Sôm (for the Sôm was evidently, the moon). According to it, Sôm is illumined by rays coming from the sun, during the new-moon-period. And, of course, an appreciation of the fact that it was the solar rays that light up the moon, was a great step forward, towards an explanation of an eclipse. [Art. 12.]

6. Further, the Chaldeans who had apparently made a wonderful progress in Astronomy, long before the days of Greek civilisation had discovered the saros (lit. repetition). This consists of 223 lunations in a period of about 18 years, 11 days constituting a cycle and includes an exact number of periods of the revolution of the moon's nodes, relative to the earth ; and the Chaldeans had found that eclipses during one cycle are *repeated* during the following cycles, exactly in the same order and almost under similar circumstances. In order to understand this, it is necessary to remember that an eclipse takes place, when the sun, the earth, the moon and the node of its orbit are very nearly in a line. And it is easily seen that all the configurations that satisfy this condition—for an eclipse—will continually recur in a fixed sequence, at every succeeding saros, on account of the virtually unchanging character of the motions of the earth and the moon.

7. But perhaps the explanation of the remarkable property of the saros was not known. It was most probably derived as a generalisation from observation alone. In order to arrive at this generalisation, however, it was, obviously, necessary to observe and tabulate the eclipses for a considerable length of time and with an accuracy, which presupposes a high standard of Astronomical

knowledge. In fact, we are told that Chaldeans had prepared star charts and begun to use the Signs of the Zodiac for determining the course of the sun, the moon and the planets and appear to have used these star charts for navigation, so that the discovery of the saros was really only one—though perhaps the highest—of their achievements, in Scientific generalisation.

8. It is difficult to say, who were the first to use the Signs of the Zodiac. The Hindu Astronomers used two systems of reckoning; the lunar mansions or *tithis* and the signs or the *rashis*, the first being obviously the earlier of the two. For,[1] while the moon's motion among the stars is a matter of direct observation, the solar motion, in its relation to the stars could only be observed by an indirect method, on account of the fact that his light shuts out of view, all stars in his neighbourhood. On the other hand, the moon's motion is much more irregular than that of the sun, while further, it is by the sun's motion, mainly, that our daily lives are ordered. The observation of the sun's motion, therefore, came gradually to be recognised as a matter of practical as well as of scientific importance and the method of signs or *rashis* ultimately superseded the method of the *tithis*. We may thus be sure, *a priori*, that the lunar system gradually led up to the solar. As to the lunar system of the Hindus, its high antiquity is testified to by the fact that the primitive series opened with *Krittika* (the Pleiades) as the sign of the vernal equinox. But this arrangement would be correct only about 2300 B. C. and "nowhere else would be found a well-authenticated Zodiacal sequence of so early

[1] It is noteworthy that most writers on Ancient Astronomy are not professed Astronomers. They have accordingly failed to take note of points which would naturally appeal to those who have to do with practical Astronomy at first hand.

a date."[1] If this is granted, it seems to be very probable that the method of signs was built up in India, for the method of *tithis* which is admitted to be peculiar to India —(at any rate, not derived from the Greeks [Art. 9]) may be regarded as the parent of the method of signs, and we are thus able, almost, to trace a gradual evolution of this system.

9. Whether this view of the genesis of the Zodiacal signs can be substantiated on direct evidence is a matter which has not been properly investigated, though there has been much discussion, of little importance, from a strictly astronomical point of view, as to who the real originators of the Zodiacal systems were. Biot regarded the Chinese *sieu* as indigenous *and as a necessary consequence*, the Hindu *Nakshatras* and the Arab *manazil* borrowed from the Chinese. Professor Weber has, however, proved that the Chinese *sieu* as well as the Arab *manazil* in respect of order, number, and identity of limiting stars correspond to a *later* phase of Hindu Astronomy, which has a distinct History of its own, prior to that phase. He adds that the Hindus seemingly founded their lunar mansions which the Arabs borrowed. In this, Professor Weber is supported by Colebrook. But Professor Weber has propounded the view that the Zodiacal system originated in Babylon and this view has been accepted (with some hesitation) by Professor Whitney. Such a view, however, can no longer be maintained, since we have now to admit that Babylonian system is based on the sun's motion. This being the case, if the view adumbrated above that a lunar system must be of an older date than that which is based on the sun's motion is correct, and astronomical arguments

[1] *Encyclopædia Brittanica*, art. 'zodiac.'

clearly point to its verisimilitude, we must admit that the Babylonian system itself was derived by a process of adoption. It is conceivable, of course, that the different systems—Indian, Chinese, Babylonian and Greek may have grown up side by side, although Astronomical arguments point to only one process of evolution.

10. Whoever the first inventors of the Zodiacal systems may have been, this device of the signs and asterisms seems to be of remote antiquity and speaks volumes for the ingenuity of the early students of Astronomical Science. In modern times, with our fixed observatories, our instruments of precision—clocks and transit instruments, it is a comparatively simple matter to determine the position of the sun or any other celestial body, at any time. We have, in fact, only to note the moment of the transit of the body across the meridian of a place, by means of the sidereal clock. This gives one co-ordinate and the altitude of the body at its meridian passage gives the other co-ordinate; and by means of these, it is possible to accurately represent the position of the body and its motion at any time. The ancient Astronomers had no such means at their disposal. They early recognised that the various groups of stars or constellations seemed to be bound together by an invisible chain and to be apparently fixed or practically so, to the celestial dome or vault, which appeared to rotate about a certain definite axis, practically fixed in space.[1] They must have noted, in the next place that there is one family of constellations arranged along the whole of the celestial region, through which the sun, the moon and the planets (known to them) pursued their course. This family of constellations might, therefore, well be used and came ultimately to be used, like

[1] Cf. *Suryya Siddhanta*, Ch. XII, 55.

so many sign-posts, for the purpose of indicating and describing the positions and motions, of these bodies.

11. We may state the argument in a different form: Modern Astronomy teaches us that the sun's path on the celestial vault is a circle. If this is suitably divided into twelve parts, each arc will be found to be occupied by a group of stars, called a sign of the Zodiac-irregularly paced no doubt, but so, that the group may be taken roughly to give a distinctive character to the particular sub-division which it occupies. Starting with any point of time of reckoning, say from an equinox (that is the moment at which the sun is at the equator or when we have equal day and night, throughout the earth), each of these signs will be passed over, roughly, in one month [1] (or one-twelfth part of a year) and one mode of describing the sun's motion would, obviously, be to name the particular sign and the position in that sign that the sun occupies, at any particular epoch. In the same way, the lunar path in the celestial vault being also a circle, this path might also be used in the same way, as the circle of reference. This latter circle (as well as the former one) was used by the Hindus who divided its circumference into 28 parts or, as it was done later, 27 parts, and called each arc a *tithi* or a *lunar mansion,* to which reference has already been made. Now, as the sun's path as well as the moon's are contained within the same belt of the celestial vault, the solar as well as the lunar positions might be described with reference to the *tithis* as well as to the signs. The former would provide a more accurate, if a somewhat less convenient [Art. 8] description. A division of the circular path of the sun into 365 parts, as in Chinese

[1] *Suryya Siddhanta,* Ch. I, 13.

Astronomy or 360 parts, which is now generally accepted and was used in *Suryya Siddhanta* (Ch. I, 20) would mean a daily description of nearly one of these divisions in one day. These latter, together with further sub-divisions which are being constantly refined would obviously make for increased accuracy but the advantage of the Zodiacal system on which the heavens themselves furnished the dial-plate, and the sun [or the moon] itself served as indicator of the day and the month, in ages which had not as yet perfected the geometrical and the instrumental methods of later times cannot be over-estimated. And the fact that the motions of the sun, the moon and the planets are all confined to a narrow belt with the ecliptic (or the circle defining the path of the sun), as the central line, enhanced the usefulness of these modes of representation.

12. When the motion of the sun and the moon became completely known and their posititions could be predicted, the calculation of the eclipses was naturally the next stage in the evolution of accurate Astronomy. For observations, such as those which were the basis of the Zodiacal system and on which the saros was ultimately constructed very early led to the inference that, in the matter of an eclipse, the positions and motions of the sun and the moon (as actually observed) were the determining factors and the problem was attacked and ultimately solved by the ancient astronomers on the supposition that the observed motions were also the real motions. The solution obtained, however, was correct, though naturally not as accurate as modern methods will yield. For, it should be noted that for a successful solution of the problem of an eclipse, it was not necessary to definitely grasp the fact that it was the earth that was in motion ; the result would be the same, if the earth were

at rest and the sun was moving about it, as the phenomenon is dependent on the motion of the cone of shadow cast by the earth, relatively to the moon and this motion would be the same (except as regards direction), whether the sun is at rest and the earth, in motion or *vice-versâ*.[1]

13. Observation of the sun's motion with reference to the Signs of the Zodiac must have very early led to the discovery of the phenomena, not, probably, of course,—the explanation—of the precession of the equinoxes—the fact, namely, that at each succeeding equinox, the sun does not come back to the same star, but that the signs and therefore all stars are observed to have a motion relative to the point, which the sun occupies at either equinox and that the direction of motion is opposite to the sun's observed [annual] motion among the stars. Hipparcus (134 B.C.) was led to this discovery, on observing a star which was new to him, but the precession was apparently, according to some, long known to Hindu Astronomers, perhaps before 1192 B. C. and its rate determined by them—necessarily, only roughly.

14. In a history of Astronomy, however brief, the subject of precession deserves more than a passing reference, for the discovery of precession was essential to the progress of accurate observational Astronomy; the subject has, moreover, an added interest in that by taking account of precession, we are enabled to ascribe dates to recorded observations, as well as to past events which can be associated with the prevalent Astronomical knowledge of the times, at which they occurred. A few words in explanation, therefore, may not be out of place here.

[1] Thus, the *Suryya Siddhanta* states the geometric aspect of the phenomena quite accurately.

15. The path of the sun in the celestial vault being accurately a circle [Art. 11], it follows that his orbit (assuming the sun to be in motion) must be a *closed plane curve* and observation of stars which may be regarded as fixed on the celestial vault (and in space) leads to the conclusion that this plane is fixed (or nearly so) in space. The line perpendicular to this plane, through the centre of the celestial vault is, therefore, fixed in direction (in space) and precession consists in the rotation of the earth's axis about this line in a period of about 26,000 years. The point at which the polar axis meets the celestial vault thus describes a small circle in space and, as a necessary consequence, the stars that occupy the region marked by this circle become pole stars in succession. While this goes on, the line of intersection of the equator and the ecliptic (which passes through the sun at an equinox) points to different stars at different epochs; in other words, in consequence of precession, the sun occupies at an equinox, different signs at different epochs. The motion of the earth to which precession is due is in fact similar to that of a top, spinning with its axis inclined to the vertical, the axis of the top, corresponding to the earth's axis, the vertical, to the axis of the ecliptic and the spin, to that of the earth (to which the apparent diurnal motion of heavenly bodies is due). And it may be added that the dynamics of the motion of the top is the same as the dynamics of the earth's motion.

16. It is this precession, among other causes which led to a great confusion in the matter of the calendar. Defining the year, in general terms, as the period in which the sun completes its cycle, it is easily seen that the cycle may be said to be completed, either when the sun returns to the same point of space—the same point of a *tithi* or a *rashi* or to the same solstice or same equinox, as the one from

which the year is reckoned. The first is called the sidereal year, the second, the tropical year. Or, again, the year may be determined by moon's motion[1] or that of any other celestial body. Add to this, the difficulty in arriving at the length of any of these periods, both on account of the difficulty of observation, as well as to the fact that none of these contain an exact number of days and that the day (the solar day) is not a constant interval of time and our wonder is not that the problem of the calendar could not be completely solved till quite modern times, but that so much was accomplished in this direction in India, China, Chaldea, Egypt and Greece in ancient times.[2]

17. Ancient Astronomy and much of modern Astronomy is necessarily observational, dealing with Astronomical phenomena—mainly, motions of celestial bodies—as they appear to the observer. It is on accurate observations alone that any scientific generalisation could be based but these were not easy, before the days of the clock and the telescope. Yet, we have seen how most remarkable results were obtained even in distant ages. The high-water-mark, however, of observational Astronomy, so far as it could be perfected without the help of the telescope or the clock was reached by Tycho Brahe (born 1546 A.D.) who built a splendid observatory in the island of Roskild under King Frederick of Denmark, fitted with the armillary sphere,

[1] Cf. *Suryya Siddhanta*, Ch. I, 35.

[2] To consider the manner in which this and other Astronomical discoveries were made as well as the question of priority among different nations would be a task, difficult of accomplishment. It would, however, be an interesting and fascinating inquiry which can only be successfully carried out by a professed student of Astronomy. The difficulty of the task is enhanced by the fact that it will be necessary to eschew all personal bias [Art. 2], to remember that science is neither of the East nor of the West and to rigidly adhere to it, as a cult.

sextants, mural quadrants, celestial globes, etc. Inspite of the inherent imperfections of these instruments, he prepared a remarkably accurate catalogue of 1000 stars and an accurate table of refractions. He also proved that the stars and comets had no annual parallax; that is, the angle subtended at any—even the most brilliant—star by the diameter of the sun's apparent orbit was infinitely small and, that, accordingly, all the stars must be very far off. He also obtained more accurate results regarding the moon's motion, than were known before his time and a considerable body of most accurate data, regarding the motions of the planets, specially that of Mars.

18. It was these accurate observations which, in the hands of Kepler led to a complete solution of the problem of the real motion of the planetary system. *Primâ facie*, it was natural to attempt an explanation of the observed motions of the planets, on the postulate of a stationary earth. But to explain—that is, to give a coherent account of the motions of the planets on such a postulate seemed to be well-nigh impossible. Among the first attempts at analysis were those made on the dictum of Plato (427 B.C.) that the circular motion was the perfect motion, and for 2000 years, astronomers who accepted the Platonic dictum attempted to represent planetary motions by means of circular and epicyclic motions. A point on a circle, the centre of which moves on another circle, their concavities being turned in opposite ways describes an epicycle, the actual nature of the curve depending on the relative lengths of the radii of these circles. If the second circle also moves, we have an epicycle of a higher order and so on. A further complexity in the motion can be introduced, if the moving point does not lie on the circumference. The object of Mathematical astronomy from the time of Ptolemy was to imagine suitable combinations of circles

which would adequately represent the observed planetary motions but such attempts proved to be obviously futile, for the attempted explanations were as complex as the motions themselves, which they could never fully represent.

19. Johan Kepler was an assistant of Tycho Brahe and came into the possession of the latter's splendid results after his death. It was after vainly attempting to fit in these results, the accuracy of which was undoubted with a hypothesis of epicycles of increasing degree of complexity, that he gave up the postulate of the stationary earth and adopted the hypothesis of a moving earth—moving about the sun. The conception was not absolutely new. It would, however be difficult to say when it was first propounded. The Hindus knew that the motion of the planets could not be explained by circular motion round the earth.[1] Pythagoras (569-470 B.C.), who according to some, came to India to study Mathematics also propounded a system, somewhat similar to that finally adopted by Kepler but he had offered no grounds for such a theory. The idea was revived by Copernicus in the 16th century but, as he also attempted to explain all motions as made up of circular motions, his theory failed to justify itself. Kepler's work therefore stands alone, in as much as he not only postulated a helio-centric system but from a detailed analysis of the results of Tycho's observations, specially on the motion of Mars showed that the orbits of the planets are ellipses, variously inclined to each other and to the ecliptic (that is, the plane of the earth's orbit), with the sun at one of the focii. He went further, for he deduced from these observations his three

[1] It has been noted by Bapudev Sastri that the planetary motions given in *Suryya Siddhanta* were those round the sun. But this might have been used along with the hypothesis of a fixed earth [Art. 26].

celebrated laws which embodied a remarkably complete and coherent scheme.

20. These laws may be described as

(1) The law of equable description of area;
(2) The law of elliptic orbits;
(3) The law of periodic times.

Stated in the form in which they were given by Kepler, they appeared in spite of their generality to be extremely artificial, like many an empirical law of modern science, the rationalé of which is unknown. Specially is this the case with the third law—that the squares of the periodic times vary as the cubes of the major axes of the ellipses described by the planets. For it seems to recall the law enunciated by Newton in his optics that the "length of a *fit* varies as the secant of the angle of incidence." Thus, the very statement of these laws seemed to demand an inquiry into the nature of the simpler law, from which they are deducible.

21. Kepler himself was alive to this point of view. He not merely laid down these laws as deductions from observed results; he saw that everything pointed to the sun as the centre of the planetary system, in a *dynamical* sense. He was, in fact, clear in his views, regarding the principle of universal gravitation which he saw, was operative in this case. The principle itself, however, was known long before Kepler. It is conceivable that the germs of the principle are traceable to early thinkers[1] just as the atomic theory can certainly be traced to Lucretius;

[1] Such attempts were necessarily mere speculations, before a coherence was established in the midst of the somewhat chaotic data that were alone available.

There is one noted passage in *Suryya Siddhanta* (Ch. II, 9) which may be given a dynamical interpretation.

"The attraction on the sun is very small by reason of the bulkiness

but, in any case, the principle is given by *Varahamihir*, in the 6th century. He wrote :—"The earth attracts that which is upon her." It is, also, given by *Brahmagupta*, in the following more complete form: "All heavy things fall down to the earth by a law of nature, for it is the nature of the earth to attract and to keep things, as it is the nature of the water to flow, that of the fire to burn and that of the wind to set in motion." But if the principle was not new, it remained barren of results, though Kepler fully appreciated its importance. It was, thus, left to Newton to develop its remarkable significance, even if he did not rediscover it.[1] Starting from first principles—from the laws of motion, which had been previously discovered by Galileo and others, he showed how the laws of Kepler, regarding planetary motions were the consequences of the sun's attraction, directed to its centre, on bodies projected with initial velocities of suitable magnitude. He explained how the somewhat complicated motions of the moon are to be accounted for as the resultant of the action of the sun and the earth and the neighbouring planets, that precession and nutation (*i.e.*, the periodic variation in the inclination of the ecliptic to the equator) is also similarly due to the action of the sun and the moon on the bulging portions of the earth, that tides are caused by the same action and that the figure of the earth itself is due to the mutual gravitation of its parts and the effect of its rotation about an axis.

of its body but that on the moon is greater than that of the sun. on account of the smallness of the moon's body."

This may be interpreted as the gravitational law regarding masses of bodies, if we read 'acceleration' for attraction.

[1] The story of the apple is not well-authenticated. Still it illustrates a truth, which is now being gradually recognised that all great discoveries in Science, as in everything else are, in reality, inspirations of genius.

22. Newton applied the law to the comparison of the masses of the heavenly bodies and, altogether, his investigation completely demonstrated the principle that the sun is the ruler of a dynamical system, obeying one simple law, and later investigations have, in every case, only confirmed this principle.

23. The mathematical method of Newton enabled Halley to deduce from recorded observations that the comet, he had observed in 1682 moved in an elongated ellipse, differing little from a parabola and he was thus able to predict its re-appearance. When account was taken by Clairaut of planetary perturbations, he was able to predict its perihelion passage (*i.e.*, the nearest approach to the sun), on the 13th of April 1759. The comet actually reached perihelion on March 13th, 1759. The calculations of Cowell and Crommelin have supplied a still more accurate knowledge of its orbit, which its last appearance has abundantly confirmed.

24. After the discovery of Uranus, it was observed to deviate from its calculated path, in a manner which suggested that the deviation was caused by the disturbing action of an unknown planet. From known perturbations, it was possible on the mathematical method of Newton, to determine the nature and position of the unknown planet, assumed to produce the disturbance. The problem was solved, simultaneously by Adams and Leverrier and subsequent observation confirmed completely the deduction of theory, by the discovery of Neptune. This was a most signal triumph of Newtonian theory which now justly ranks as an indisputable scientific truth.

25. An account of the application of dynamics to astronomy cannot be complete, without a reference to the remarkable method by which Foucault supplied an ocular demonstration of the fact that the earth rotates about

an axis, completing a cycle in 24 sidereal hours. It is difficult to say when it first dawned on astronomers that the steady rotation of the celestial vault with the stars apparently fixed on it could be simply explained on the supposition of the earth's rotation. A clear statement of this principle is to be found, however, in *Aryyavatta.* He says that "the stars are fixed; it is the rotation of the earth that causes the daily rising and setting of the stars." It must be admitted, however, that a demonstrably exact knowledge of this point belongs only to the last century, based on Foucault's researches. He showed that the axis of a gyrostat which is set spinning always points to the star to which it is initially directed and that a long pendulum which continues to swing for a long time, appears to change its plain of oscillation at a known rate. These lead to the conclusion, as Foucault proved, on simple dynamical reasoning, that stars occupy fixed positions in space and that the earth rotates about an axis, round which the stars appear to move, while pursuing their apparent paths.

26. In the same way, anything like a direct proof of Kepler's hypothesis of the earth's motion round the sun belongs also to the last century, when the discovery of aberration by Bradley led to a simple demonstration of its truth. Bradley discovered, that, when star-places were accurately observed, they appeared to describe small ellipses about their mean position, parallel to the ecliptic and to complete a cycle in a year. This can only be due to the motion of the observer, that is, of the earth carrying the observer with it and we have a practically ocular demonstration of the earth's orbital motion.

27. Until, however the genesis of the solar system has been investigated and gravitation itself explained, Science cannot be said to have done more than take the first step towards the elucidation of the mystery of

the laws, in obedience to which, the heavenly bodies pursue their appointed course.

28. Accordingly, scientific men now-a-days are not content with the advance, already made. The next step in scientific generalisation remains to be made, *viz.*, to explain the law or principle of gravitation itself. The law, as we now know it, is obviously artificial; it states that every body or every particle attracts every other body or particle according to the law of inverse square of the distance. There is no reason, obviously, *a priori* why the distance law or the index law should hold. It has accordingly been suggested that there must be some more fundamental principle underlying that law, and science is making a bold effort at getting at that fundamental principle. A simple illustration will suffice. It is well-known that if two spheres are made to move along the line joining their centres with constant velocity in a fluid medium, they appear to repel each other. This repulsion is an apparent repulsion due to the peculiar property of motion in a fluid medium, and it is conceivable that something of a fundamental nature connected with the property of the medium in which the so-called gravitational law operates is at the bottom of the peculiarity of the law of attraction according to the inverse square of the distance. Again, it has been found possible to suggest an explanation of the energy of the mutual action between bodies as due to some subtle motion of the medium in which these bodies are placed and it is not unlikely that this subtle motion of the medium may also operate in producing the effect which we describe *provisionally* as being due to gravitation.[1]

[1] It is on a conception of this kind that the theory of relativity has supplied a complete explanation of gravitation including a modification of the Newtonian theory required to explain many outstanding phenomena.

29. Then, there is the question of the genesis of the solar system. Newton has shown that the nature of the orbit, elliptic, hyperbolic or parabolic depends on the initial velocity of projection of the moving body. And the question is pertinent as to the nature of the explosion which generated the initial velocity—the 'velocity of projection' of the requisite amount. It is, moreover, worthy of note that nearly all the planets and satellites are coursing the same way round, as well as rotating about their axes in the same direction. To one line of speculation, to explain all these, we shall confine ourselves.

30. Matter, as a nebulous mass, diffused through infinite space, would on account of mutual gravitation of its parts tend to be gathered together into a body like the sun, whose potential energy of shrinkage may account for the energy which the sun is known to lose through radiation. If we further suppose that the nebulous mass had originally a motion of rotation about an axis or,—for some want of symmetry—to acquire such a motion, it would tend to have increased angular momentum (on the principle of conservation of angular momentum) and as a result may give up rings and each ring may gather up into one or more planets, themselves rotating about their own axes.[1]

[1] This hypothesis seems to have received some confirmation, since the photograph has revealed the remarkable behaviour of spiral nebulæ in Andromeda, which almost presents to the eye the process indicated in the above hypothesis. Mathematical calculation has shown, however, that a rotating liquid or matter of high density will break up into binaries and triplets, while a rotating mass of gas of extreme tenuity will assume gradually a lenticular shape and give up, not rings but filaments at the two extreme points of the lense, which will break up into nucleii, regularly spaced out.

The hypothesis seems, thus, capable of explaining the formation of binary and triple stars as well as spiral nebulæ giving up star clusters. Whether it is capable of explaining the solar system, also,

31. For the materials of each ring would continue to cool and contract from the gaseous to the liquid condition. If the contraction were uniform, the ring would break up into a large number of small planets. On the other hand, if it is not, and this will be the case generally, on account of some want of symmetry that we must postulate in the medium, some parts will condense more rapidly than others. The effect of this will be to form a single mass—a single planet. In the same way, satellites will be formed from the nascent planets.

32. Again, we know that the moon causes tides on the earth—on the oceans as well as on the solid earth. It has been shown that tides caused by a satellite on its planet or *vice versâ* gradually cause changes in the relative motions of the pair and in their distance apart. From this, it has been concluded that the moon separated from the earth about 57 million years ago—assuming that this cause alone has been operative and that in course of time, the earth's period of rotation will become a month, as it is already the period of rotation of the moon.

33. But quite apart from these speculations, the planetary system, as we know it now, presents a simplicity, which is indeed remarkable, in view of the complexity which it presented to ancient astronomers. Side by side with this simplification, we have to place the achievements of the telescope, the spectroscope and photography which have brought a new world within our purview. Whoever the actual inventor of the telescope may have been, it was Galileo who constructed the first instrument used in Astronomy. The telescope helps in two ways. It enables the Astronomer to observe along a mathematically defined line—the

cannot, as yet, be stated for certain, for a gas under its mutual gravitation cannot be homogeneous, but the corresponding mathematical problem still awaits solution.

optic axis of the telescope,[1] while the magnification produced by it has revealed the wonders of the heavens previously unknown and unsuspected. Thus, the physical features of the moon and the details of the surfaces of the planets, can now be examined in detail, while it has added considerably to the list of planets—not merely Uranus and Neptune, but also a large number of very small planets between Mars and Jupiter, called the Asteroids. We also owe to it the discovery of a considerable number of satellites of the various planets. It has also in some cases resolved stars (*e.g.* capella) into *binaries*. But the telescope has also revealed the immensity of the heavens. We now know, that, however much our powers of observation are sharpened, the stars must ever remain to us mere points—so immensely distant are they and that the solar system is separated from other starry systems by distances, which can only be described as immense, as compared with the magnitude and the dimensions of the solar system, itself.

34. There is a limit, however, to the power of the telescope imposed by its own mechanism as well as man's power of vision. Accordingly, for further information on the celestial bodies, we must resort to less direct methods. The spectroscope has enabled us to obtain definite information, regarding the constitution of the sun and the stars, of nebulæ and of double and variable stars. And this, together with photography has led to the discovery that the so-called fixed stars are in reality in motion. Photography has also enabled the astronomer to map out the heavens and study its contents at leisure.

[1] At the *Man Mandir* at Benares there is a hollow tube of small bore capable of being pointed in different directions which was evidently used to fix the direction of observation.

35. The stars and nebulæ which are too faint for the telescope, as well as the details of celestial bodies that would have remained undeciphered through its means reveal their existence on the photographic plate which has thus become indispensable to a more thorough survey of the heavens than what the telescope could alone have attempted.

36. Again, the evidence of the spectroscope conclusively proves that stars are self-luminous bodies like the sun. They are, in fact, so many distant suns with, therefore, it may be presumed, systems of their own—which it would be quite impossible for us to become cognisant of, with the means, at present at our disposal. If, therefore, each star with its system is separated from its neighbour by a void as great as ours is, we have some slight glimpse of the vastness of the starry heavens and of the incompetence of man to survey its infinite magnitude.

37. Thus, our most recent discoveries have but led to the conclusion that we can, by the most refined means at our disposal, learn but extremely few of the secrets of the starry heavens, so that we, in the twentieth century, are, after all, but little removed from those, who, in the infancy of the human race, looked with wonder on the enchanted—if sublimely mysterious—universe around them and if they prostrated themselves before a Supreme Being who to them seemed to be the solution of the mystery,[1] let it not be said that we have less imperative reason for doing the same, although we have been permitted to see a little—but very little, behind the veil.

[1] Cf. *Suryya Siddhanta*, Ch. I, 1.

# CHAPTER I

## THE SPHERE

1. *Def.* If a semicircle (ADC) (fig. 1) is made to rotate about the bounding diameter, the solid generated is called a sphere.

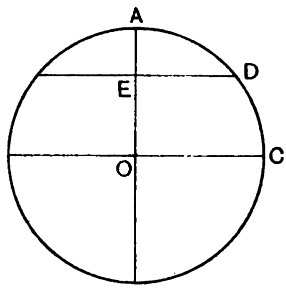


Fig. 1.

2. From the mode of its generation, it is evident that:—

(1) If O be the centre of the circle, the distance from O of every point on the surface is the same and is equal to OC, the radius of the circle.

This distance is called also the *radius of the sphere.* And all lines passing through the centre and terminated both ways by the sphere are its *diameters.*

(2) The semi-chords, such as ED, as well as the radius OC sweep out circles which are all perpendicular to the bounding diameter and are, therefore, parallel to one another.

3. Now, it is evident that the radii of all such circles except that described by the radius OC are smaller than the radius of the sphere. These circles (having the semi-chords ED, etc., for radii) are called SMALL CIRCLES.

*Per contra*, the circle having OC for its radius is called a GREAT CIRCLE. And, generally, any circle that can be drawn on the sphere whose radius is equal to the radius of the sphere, is a great circle.

Thus, the circle ADC, and all the circles with which it coincides during its rotation are great circles.

It is evident that all great circles on a sphere have for their common centre, the centre of the sphere.

4. We have, thus, two sets of circles. *Viz.*, (1) the great circles of the type ADC, and (2) the set of circles (parallel to one another) perpendicular to the diameter about which the semi-circle ADC is rotated.

*Def.* The line, about which a figure is rotated is called the *axis* of rotation.

These two sets are evidently perpendicular to one another. Their relative positions may also be described by saying that the first set contain the axis, while the second are perpendicular to it.

5. With reference to this axis,

the first set are called MERIDIANS OR SECONDARIES to the second set, while of the second set, the one (a great circle), passing through the centre is called the EQUATOR, and the rest, PARALLELS.

6. *Def.* The either extremity of the axis is called a pole of the equator (*e.g.* A, fig. 1) and it is clear that all meridians pass through the poles.

7. From the symmetry of a spherical surface, it follows that any other diameter may be regarded as the axis and we shall get with reference to this axis, again, two systems of circles, *viz.*, (1) meridians, (2) equator and parallels. Hence, all meridians are perpendicular to the corresponding equator.

8. If a plane is drawn through the centre of a sphere, it will intersect the sphere along a great circle. For

the curve of intersection is necessarily a plane curve, every point of which is at the same distance from the centre.

In other words, a central plane section of a sphere is a great circle. Any such great circle being regarded as the equator, it follows from art. 2, that all parallel plane sections must be (parallel) *circular* sections or *its parallels*.

This can also be proved directly.

For, let the plane CDE (fig. 2) intersect the diameter OC which is perpendicular to the equator OBA at C. Then, CD is perpendicular to OC, where D is any point on the curve of intersection of the plane and the sphere. Let O be the centre of the sphere;

$$\text{then } OC^2 + CD^2 = OD^2 = \text{constant.}$$

But OC is const.

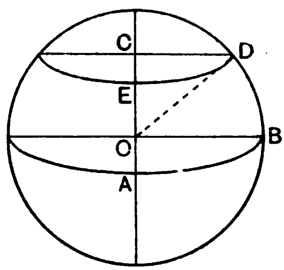


Fig. 2.

Therefore, CD is constant, or the locus of D is a circle having its centre at C.

9. It follows, therefore, that all plane sections of a sphere are circles, the *central* ones being *great circles*, and those that are *non-central, small circles*.

10. *Def.* The angular distance between any two points on a sphere is the angle subtended at the centre of the sphere by the line joining these points.

This is, evidently, also the angle subtended at the centre of the sphere by the arc of the great circle passing through these points.

For, the section of the sphere by the plane containing the radii to these points determines the great circle through them.

11. Since the angle subtended by an arc of a great circle at the centre is proportional to the arc, the angular distance between any two points may also be expressed by means of the length of the arc of the great circle passing through these points.

12. The poles of the equator are at an angular distance of 90°, from every point on the equator. Hence, all its secondaries pass through the poles. *Per contra,* the poles of the secondaries, all, lie on the equator. Thus, any two great circles at right angles to each other have this reciprocal relation, *viz.*, the pole of either is on the other.

13. Remembering that a point on a sphere may be defined as the intersection of two great circles, the position of a point S on a sphere (centre O), may be represented as follows : [Fig. 3.]

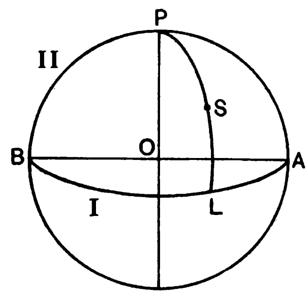


Fig. 3.

Take two fixed great circles (I and II), called the great circles of reference, at right angles to each other (*e.g.*, the equator and one of the meridian circles, [Art. 4.]

Then, P, the pole of I will be on II and if AB be the intersection of I and II, the angular distance of P from A and B will be 90°.

Now, describe a great circle through P and S. This will be perpendicular to I, *i.e.*, a secondary to I. [Art. 4.] Let it meet I at L.

Then, the position of S will be evidently determined, if the arcs AL and SL or the angles (AOL and SOL) subtended by them at the centre of this sphere are known.

For given AL, we know which of the secondaries to I contains S and when this is known, LS determines its actual position on that secondary.

*Def.* The figure bounded by portions of three great circles on a sphere is called a spherical triangle.

Obs. Any two points on a sphere may be joined by a great circle. [Art. 10.]

14. To determine the position of a place (a point) on the surface of the earth (supposed spherical) :

Now, it is known that the earth rotates about an axis, the position of which can be determined with absolute accuracy. [Ch. III, art. 15.]

Hence, although any two great circles drawn on the surface of the earth might have served for determining the position of a point on its surface, it is naturally convenient to take the great circle perpendicular to the axis of rotation as the circle of reference I. This is called the *terrestrial* equator. The secondaries to the equator are called terrestrial meridians or, simply, meridians.

The advantage of this is that this circle of reference can be determined with ease and, therefore, measurement with reference to it can be carried out in practice by direct observations.

For II, we may take any meridian that it is convenient to take. In other words, we may take any great circle passing through the poles (Ch. III, art. 12) and a suitably chosen point on the surface of the earth. This is called the prime meridian.

In English Astronomical works, the meridian through Greenwich is taken as this second great circle of reference, or prime meridian. The French use the meridian through Paris, for the same purpose.

In the Hindu Astronomical work *Suryya Siddhanta,* the meridian through *Ujjain,* as well as that through *Lanka* is used.

15. The equator AB and a suitable meridian BPA being taken as the great circles of reference (fig. 3), the arc AL (or the angle AOL) is called the *Longitude* of the place S, east or west and LS (or the angle LOS) the *Latitude,* north or south; that is

the Latitude of a place is the angular distance of the place from the equator, measured north or south, and

the Longitude of a place is the angular distance of the meridian through the place from a fixed meridian, (measured east or west). [Ch. III, art. 27.]

16. *Note.*—The angular distance of a point from a great circle is the angle subtended at the centre of the sphere by the intercept of the secondary of the great circle through the point, between it and the great circle.

The angular distance of one great circle from another is the angle between the two great circles. This is the same as the angle subtended at the centre of the sphere by the intercept, lying between them, of their common secondary.

*Def.* The angles of a spherical triangle are the angles between the respective great circles which form its sides.

**EXERCISE.**

1. Define a sphere, a great circle, a small circle, equator, a meridian and parallels.

Two great circles intersect at P and Q. Prove that the straight line PQ passes through the centre.

2. Define terrestrial latitude and longitude.

Two places on the equator have longitudes of $10^\circ$ and $60^\circ$ respectively. Find the length of the equator intercepted between them. (The radius of the earth$=4000$ miles.)

3. Prove that all plane sections of a sphere are circles. Two planes passing through the centre are inclined to each other at $45^\circ$. Find the length of the secondary to their circles of intersection with the sphere, intercepted between them.

Prove that the line of intersection of these two planes is perpendicular to the secondary.

4. A, B are two places on the surface of the earth, whose latitudes and longitudes are given; show by means of a diagram, the direction of B as seen from A.

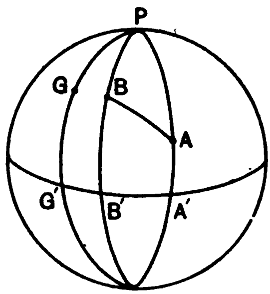


Fig. 4.

Let PG be the meridian through Greenwich,

PBB′ the E. meridian through B and PAA′ that through A, P being the north terrestrial pole and G′B′A′, the terrestrial equator.

Then G′B′ represents the E. longitude of B, B′B, the N. latitude of B.

Also G′A′ is the E. longitude of A, AA′, the N. latitude of A. Hence A, B are determined. Now, describe a great circle through A, B.

Then, the angle PAB or the angle between the meridian PAA′ and the great circle BA is the inclination of the required direction to the Geographical north. [Ch. II, art. 15.]

5. Two places on the same latitude differ in longitude by $30^\circ$. If the common latitude is $60^\circ$, find the distance between the two places, (measured along the common parallel.)

The radius of the small circle or parallel of latitude on which the places lie is $r \sin 30^\circ$.

Therefore, the circumference of the circle is $2\pi r \sin 30^\circ$.

Hence the arc of the small circle intercepted between the places

$$= \frac{30}{360} \, 2\pi r \sin 30^\circ = \frac{\pi}{12} r.$$

# CHAPTER II

## Celestial Bodies

1. Astronomy, as its name implies deals with the laws of stars or of heavenly bodies generally. These include—the sun, the moon, certain bright tracts of light in the sky, and a large (seemingly infinite) number of small bright objects, apparently embedded on the celestial vault.

2. Watching the heavens on a clear night, we easily recognise that the numerous bright objects with which the sky is bespangled fall into two main groups:—(1) Those (and these form the majority) that appear to move together, as if connected with each other by absolutely invariable bonds and (2) those (very few in number) that appear to have more or less irregular motions among the first group and are thus easily distinguished from them

3. The celestial bodies of the first group are the **stars** (proper); of the celestial bodies of the second group, some shoot down to view from the upper regions of the sky and then disappear but ultimately fall on the earth. These are called **meteors**. At certain times, they appear in swarms for a few hours, apparently radiating from one point in the sky. The most noticeable of these showers are those occurring in November and August. These evidently belong to trains of meteoric stones (coursing round the sun), which the earth encounters periodically. These stones being attracted into the earth's atmosphere become incandescent and appear as radiant points.

4. Of the rest, some grow bigger in size, as we watch them, night after night, having ultimately the appearance

of bright nuclei surrounded by heavy clouds and accompanied by trails of light.[1]

These are the comets (or hairy stars). They are rare visitants to the regions of the sky which are within our purview and remain within view for a comparatively short time, when they do appear.

5. There remain now only a few star-like bodies which seem to possess highly irregular motions among the stars, appearing sometimes to go in one direction, sometimes in the opposite direction, while sometimes to stand still with reference to the sun altogether. These irregularities are due, as we shall presently see, to the fact that they are going round the sun, like the earth. These are the **planets**. Some of these are easily distinguishable bright objects on the sky. Others are visible only through a telescope.

This name was given by the Greeks to these bodies, as well as to the sun and the moon on account of their seemingly irregular motion among the stars. (The word "planet" literally means a wanderer.)

6. If we are limited to naked-eye observations alone, there is no difficulty in recognising the sun, the moon, the comets [2] and the meteors among heavenly bodies and distinguishing them from the rest by their marked characteristics. Of the remainder, some (of those which are visible at all to the naked eye) are observed to change their positions relatively to the rest, when the observation is carried on for several days together. These are the planets. The rest appear to move together (art. 2). No further differentiation is possible without instrumental appliances.

[1] When they are actually visible to the naked eye.

[2] In the case of a few, even without the help of a telescope.



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For this purpose, we have to have resort to the telescope in the first instance. With its help, we are enabled to distinguish between the different classes of heavenly bodies, further. On looking through a telescope, however,—even the most powerful that has yet been invented,—we find a star appearing still to be a mere point, while a planet appears to be a body of finite size. Remembering that a telescope magnifies objects viewed through it, we conclude that a star is situated so far off that the best magnification produced leaves its angular diameter still inappreciable, while a planet must be a celestial object only moderately distant.

8. The telescope also reveals the planets surrounded by a few smaller objects which, as we shall see later, are observed to move round them. These are called **satellites**.

Viewed through a telescope, a comet is found to reveal to the observer characteristics of a hairy star, long before it can be recognised as such by the naked eye.

9. When the telescope is pointed to the tracts of light somewhat irregularly interspersed between the stars, they are found in some cases, to be clusters of faint stars obviously more remote than their more brilliant neighbours. In most cases, however, they appear to be patches of light still. When viewed through a spectroscope, they yield spectra, characteristic of incandescent gases, consisting of bright lines only. We are thus led to conclude that they are masses of incandescent gases. They are called **nebulæ**.

10. We shall presently see that the sun is a star, that the earth is a planet and the moon, a satellite of the earth.

Astronomy, accordingly, deals with heavenly bodies which group themselves under the following heads:—

1. Stars.
2. Nebulæ.

3. **Planets and Satellites.**
4. **Comets.**
5. **Meteors.**

(1) and (2) are bodies, occupying practically fixed positions in space. The others have motions among these.

11. In dealing with them, we may (1) consider their motions, (2) try to explain these motions on Dynamical principles, (3) discuss their constitution.

The first is the subject-matter of Spherical Astronomy, the second that of Dynamical Astronomy, while the third, that of Astrophysics.

### EXERCISE.

1. The angular distance between two given celestial objects is observed to vary, night after night. What conclusion would you draw from this as to their true nature? One of these is seen to be always at the same angular distance from a third celestial object. What would you conclude from this fact?

2. Seen through a telescope, two objects appear to change their relative positions. How would you proceed to find out what they are?

3. A celestial object appears to change in size as seen through a telescope from day to day. What would you conclude as to its nature?

# CHAPTER III

## THE CELESTIAL VAULT

### *The Rotation of the Earth.*

1. On looking at the sky, on a clear night, one gets the impression of myriads of brilliant points all lying on a spherical surface. In fact, for a long time, the starry heavens (or "the universe")[1] was regarded as spherical, so that "the spherical vault of the sky" has been the accepted mode of expression by astronomers from the earliest ages. We have just seen, however, that telescopic observation leads to the conclusion that stars are infinitely more remote than the planets. It is, moreover, reasonable to suppose that the fainter stars are more remote than the more brilliant ones. The impression produced by the celestial vault, therefore, namely that of a spherical surface, must be purely subjective—due to the fact that our eyes are not competent to apprise us of the relative distances of heavenly bodies, on account of the vastness of these distances, so that we fail to distinguish them from one another.

2. This explains why the vault of the sky appears to be spherical. If, therefore, with the observer's eye as centre, a sphere is described, it will be similar to the celestial vault, as it appears to him. Such a sphere is called the **celestial sphere** of the observer.

[1] *Cf.* Aristotle's argument. "The universe is spherical, because the sphere is among bodies, as the circle among plane figures, the most perfect, owing to its unique form limited by a single surface and is the only body which during its revolution continually occupies the same space." The last property, it should be noted, belongs to all surfaces of revolution.

3. If we observe the motion of heavenly bodies on the celestial vault, we easily recognise that they all appear to move in small circles, about a common axis which is fixed in space, and that the vast majority of these complete the cycle in the same time. This interval is called a sidereal day and is nearly equal to an ordinary day of 24 hours. This axis, moreover, is directed to a point in the sky, practically indistinguishable from the position of an easily recognisable bright star called the Polaris or Pole star. [Fig. 5.]

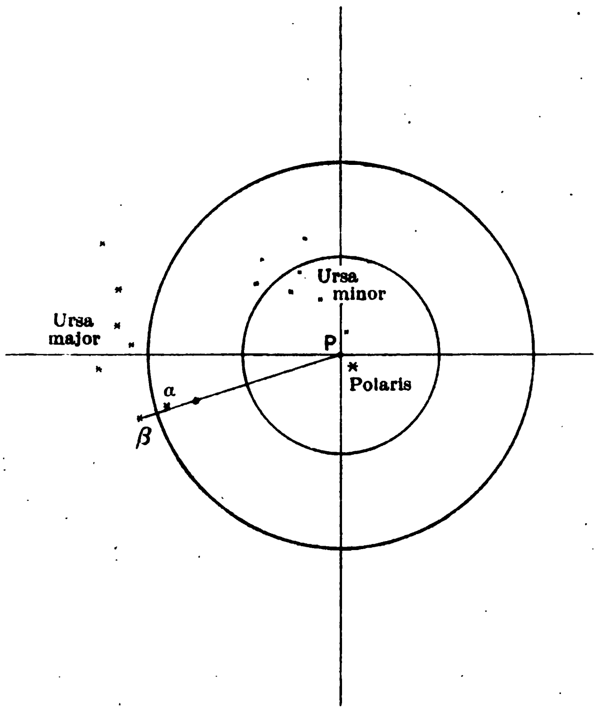



Fig. 5.

A portion of the celestial vault in the neighbourhood of polaris (flattened out), to show the relative positions of two easily distinguishable groups of stars, ursa minor, to which polaris belongs and ursa major, which include two stars α, β which are very nearly in a line with the pole.

4. If the observed diurnal motion [1] of these bodies were also their actual motion, it would be necessary to postulate an invisible and rigid bond between millions of bodies which alone can keep them always moving together. On the other hand, if the motion is only apparent and is due to the motion of the observer, we have only to admit that the earth is rotating about an axis (very nearly directed to the Pole star) completing its rotation in one sidereal day. The latter hypothesis, therefore, offers a simpler explanation of observed phenomena and, as such, should prove more acceptable, even if no direct proof were available that this is actually the case. If this explanation is accepted, we should conclude that the observed motion of stars is only apparent motion.

[1] It is difficult to say when the idea of a rotating earth was first suggested to astronomers. Both Plato and Aristotle, believed in the daily rotation of the heavens from east to west. Among Greek astronomers, however, Herakleidas of Pontus, a contemporary of Aristotle, clearly and distinctly taught that it was the earth which turned on its axis from west to east in twenty-four hours. But evidently, this idea did not gain much credence, for we find Ptolemy (2nd century A.D.), the greatest of ancient Greek astronomers, describing as a fundamental axiom of astronomy the view that "the heavens is a sphere turning round a fixed axis, as may be proved (according to him) by the circular motion of circum-polar stars and by the fact that other stars always rise and set at the same points of the horizon." At the revival of learning—long afterwards, Ptolemaic idea was the generally accepted creed in Europe till the time of Copernicus (1473 A.D.) who formulated the view that the earth rotates in a day and night. He points out that any change observed may be caused either by a motion of the object observed or by that of the observer or of both. Hence, a turning of the earth from west and east would account for the rising and setting of the sun, the moon and the stars. He, then, argues that it is very unreasonable to suppose that the immense sphere (of the celestial vault) should revolve in 24 hours and further that bodies describing smaller circles always move more rapidly than those which move in larger ones and

5. The Diurnal motion was regarded in ancient times as due to the revolution of the celestial vault—the starry sphere of the *Suryya Siddhanta*—" to the left of the Gods,[1] —as seen by observers in the northern hemisphere, looking towards the north, and to the right of the *Asuras*," and this is quite sufficient to explain the diurnal motion, if we make the further supposition that the various stars are rigidly attached to the starry sphere, regarded as a rigid dome.[2]

6. At the present day, we are not dependent on mere hypothesis for a complete explanation of the diurnal motion of celestial bodies. For this, we are indebted to a remarkables series of researches of the French Physicist Foucault, which supply an ocular demonstration of the earth's rotation.

7. It is known from Dynamics that the axis of a gyrostat which is set spinning remains fixed in space. Now, it is found that if we set a spinning gyrostat with its axis pointing to a particular star, this axis remains continually

therefore the earth must be moving—an argument, the cogency of which is not apparent. Long before Copernicus, Aryyabhatta (A.D. 476) had taught that " The stars are fixed; it is the rotation of the earth that causes the daily rising and setting of the stars " and his appears to have been the generally received doctrine in India, for many centuries before Copernicus. It would be of interest to enquire what line of argument led the Indian astronomers to this conclusion, for those of Copernicus were evidently very incomplete. But on this point, it is hardly likely that any information will be available, on account of the method of oral transmission of knowledge pursued in India in ancient times.

[1] The motion of the heavens is towards the right, because this is the more honourable direction—says Aristotle.

[2] It is unreasonable to think (says Aristotle) that each star should travel along, with precisely the same velocity as its sphere, if both were detached from each other. Therefore, the stars are at rest in their sphere. Only the latter is in motion.

directed to the same star. It follows, therefore, that the stars occupy fixed positions in space.

8. Another series of experiments [1] of Foucault's were those with a long pendulum provided with a heavy bob. If such a pendulum is set oscillating, the plane of its oscillation should remain fixed [2] in space. Observations made by Foucault and since repeated by others have shown, however, that the plane of oscillation appears to rotate about the vertical—with reference to surrounding objects at a determinate rate. This, also, easily leads to the conclusion that the earth rotates about an axis, pointing very nearly to the pole-star.

Since the plane of the pendulum is actually observed to rotate about the vertical, although it is in reality fixed in space, the observed motion of the plane

[1] Foucault took a heavy iron ball (fig. 6), about a foot in diameter and suspended it from the dome in the Pantheon in Paris by means of a string about 200 ft. in length. By means of this device, the pendulum was made to continue to swing for a long time. A circular rail having a little ridge of sand built upon it was placed under the pendulum. And a pin attached to the ball just scraped the sand, leaving a mark at each swing.

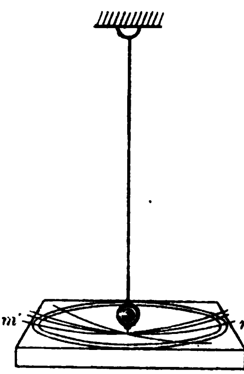


Fig. 6.

The ball was drawn aside by a cotton thread and when it had come to rest completely, it was let go by burning the cord, so that the pendulum might begin to swing in a true plane.

Inspite of all these precautions, it was found to swing in different planes at different times, according to the law, given in Art. 9.

[2] For the only forces acting on the bob of the pendulum lie in the original (vertical) plane of swing and therefore no deviation from it should result from the action of the forces that are alone known to act.

must be only apparent motion—due not to the actual motion of the plane of swing but to the motion of the horizontal plane at the place of observation.

9. Admitting, then, that the earth does rotate about an axis OP (fig. 7), let us enquire how the plane of swing of a pendulum will behave at a place Q.

Now, if O is the centre of the earth, OQ will be the direction of the vertical. [Art. 16.]

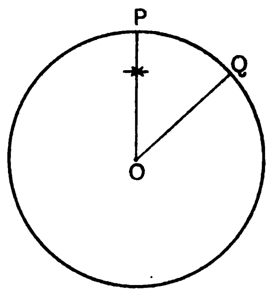



Fig. 7.

Again the angular velocity $\omega$ about the axis OP can be resolved [1] into angular velocity $\omega$ cos POQ about OQ and another equal to $\omega$ sin POQ, about an axis perpendicular to OQ.

[1] Let P (fig. 8) be a particle rotating about an axis AD with an angular velocity $\omega$, represented by the length AD and AB, AC any two lines at right angles to each other, meeting at A, Complete the parallelogram AB DC.

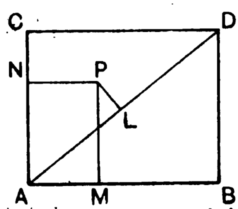



Fig. 8.

Then, we have to show that $\omega(=AD)$ about the axis AD is equivalent to $\omega_1$ $(=AB)$ and $\omega_2$ $(=AC)$ about the axes AB, AC respectively.

From P, draw PL, PM, PN, perpendicular to AD, AB, AC respectively.

The former alone will produce an angular displacement of the horizontal plane at Q about the vertical, of magnitude, equal to $\omega \cos$ POQ.

The plane of swing of the pendulum will, therefore, *appear* to rotate relatively to the horizontal plane at the same rate (*viz.*, $\omega \cos$ POQ) in the opposite direction.

Observations made by Foucault and since repeated by other observers, at different places verify this result. [1] We conclude, therefore, that the earth rotates about an axis (fixed in direction in space) with a constant angular velocity.

10. Since the earth has this motion of rotation, a celestial body must on account of this motion appear to move in the opposite direction to that of the earth to an observer necessarily partaking of the earth's motion. This completely explains the common diurnal motion of all celestial bodies. It is, moreover, found that most of these bodies appear to complete their diurnal motion in the same

Now evidently, the rectangle PM. AB = PL.AD + PN. AC.

But PM.AB = velocity (due to angular velocity $\omega$), perp.to the plane ABDC and similarly for the others,

since velocity = displacement in unit time
= product of angular velocity about A and perp. from A on the direction of motion.

Observing, now, that if the rotation is clockwise, this displacement is *towards* the observer, we conclude that the joined effect of $\omega_1$, $\omega_2$ is the same as that of $\omega$, on the same convention as to direction, throughout.

[1] Let T be the period of the earth's rotation ; then $\frac{2\pi}{\omega} = T$ and the period in which the plane of swing of the pendulum completes its cycle at any place Q will be $\frac{2\pi}{\omega \cos \text{POQ}} = T \sec \text{POQ} = T \operatorname{cosec} \text{lat.}$ (if the pendulum can go on swinging, during the whole of that time). This is actually found to be the case.

period. This period must, then, be equal to the period of rotation of the earth. This period, as we have seen, is called one sidereal day.

11. We have thus another mode of classifying celestial bodies, *viz.*, bodies which complete their apparent motion in one sidereal day and those which do not.

This leads to the following conclusions, *viz.*, that

(1) the first set of bodies are fixed in space: These are the *stars*, proper (and nebulæ),

(2) while the other bodies (*i.e.*, those which do not complete their cycle in one sidereal day) have independent motion in space, real or apparent (*viz.*, planets, etc.). [Chap. II, 5.]

12. It follows that we can determine this period (*viz.* one sidereal day) by observing the (apparent) diurnal motion of stars; for, evidently the interval between the successive transits of a star across the meridian of a place is equal to the period of the earth's rotation about its axis.

13. Now, on account of the earth's rotation, the top of a tower has a greater velocity eastwards than the bottom, as it is at a greater distance from the earth's axis. If then a particle is dropped from the top of a tower, it should not fall exactly to the bottom but deviate slightly to the east.

Thus, since the particle has initially the same velocity as the top of the tower, in the time that the particle moves to the bottom, it will undergo two displacements simultaneously. It will, therefore, reach the extremity of the diagonal of the parallelogram, having these displacements for sides. Accordingly, the particle will fall away from the bottom. The deviation will, however, of necessity, be very

slight,—so slight,[1] that the effect may well be veiled by that of the wind or other extraneous causes.

14. The diurnal rotation of the earth serves adequately to explain many natural phenomena, *e.g.*, the direction of the trade winds and monsoons.

As the equatorial regions of the earth are hotter than places in high latitudes, wind would be set up continually blowing towards the equatorial regions, both from the north and south. For the air in contact with the hotter regions about the equator will get warmer and rise up and the colder air from the colder regions in the higher latitudes will tend to take its place.

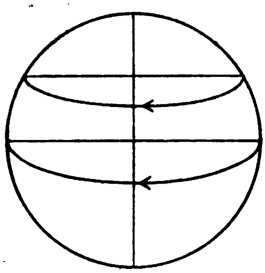
There are actually such winds but their directions are not north and south but north-west and south-west.

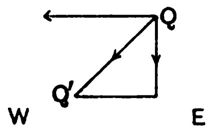
This is easily explained, as being due to the rotation of the earth. For a particle of air at a point Q at a northern latitude has lesser easterly velocity (fig. 9) than a point in the same meridian on the equator. When, therefore, such a particle reaches the equator, its easterly deviation is less than that of Q; it will, therefore, reach a point to the west of Q at Q′ (say) (fig. 10). It will result therefore that to an observer at Q′, an air particle will appear to come from the north-west. Exactly the same thing will happen to a particle of air coming from the south: It will appear to come from the south-west. Hence the trade winds are north-westerly and south-westerly.

15. **The axis of the earth.**—The rotation of the earth is thus a matter of exact demonstration. Careful

[1] Let the observation be made at a place at the equator. Then $=\frac{1}{2}gt^2=h$, if $h$ is the height of the tower = the distance fallen through in time $t$, under accl. $g$. Also deviation = $h\omega t$, where $\omega$ is the angular velocity of the earth.

$$\text{Thus the deviation} = \omega h\sqrt{\frac{2h}{g}}.$$





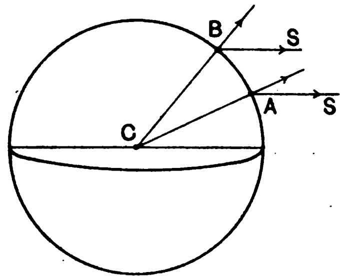




Fig. 9. Fig. 10. Fig. 11.

measurements are, however, required in order to determine the direction of this axis with accuracy. But a rough approximation can be obtained with comparative ease.

This is to follow the motion of a star with a telescope and, thus, determine the cone which its optic axis sweeps out. The axis of this cone is also the axis of rotation of the earth. It is, thus, found, as we have already stated that this axis very nearly points to the pole star. The points on the earth's surface which are situated on the axis are called terrestrial poles.[1]

The direction of the telescope in the above series of observations may be easily represented by strings or rods suitably directed. If this is done (and this was evidently the device used by ancient astronomers), the cone swept out by the telescope or line of sight can be easily materialised and the axis of rotation of the earth, roughly determined.

Modern instrumental and observational methods obviate all this labour and yield, necessarily, much more accurate result.

[1] Recent investigations have shown that these are not fixed on the earth but slightly change their position.

16. **The shape of the earth.**—We have already stated that the earth is or is very nearly a sphere. That it is not flat but globular in shape, is known from various well-known facts, *viz.*,

(1) that the earth has been circumnavigated;

(2) that the appearance of ships receding from or coming towards ports (*viz.*, funnels appearing first and disappearing last) are such as can only be accounted for by supposing that there is always a bulging portion between the observer and the ship.

That it is not merely globular but practically spherical is deducible from the observed fact that the shadow of the earth, cast on the moon at a lunar eclipse has *always* a circular outline.

This is also proved from direct measurement, based on astronomical observations.

Obs. The horizontal plane at any place is evidently the tangent plane to the surface of the earth at the place.

Hence, if we assume the earth to be spherical, the vertical direction at any place will be that of the diameter of the earth passing through it.

17. This being premised, we are in a position to find the shape of the earth.

For this, take two stations in the same meridian. This can easily be done, for two places are in the same meridian, if it is apparent noon at both places at the same time (Ch. XIII).

Now, observe at these two places (say A, B) the inclinations (fig. 11) of the direction of any, the same star to the vertical at these places, when the star (S) is on the meridian.

As S is very far off, AS and BS being in the same plane are parallel to each other.

Then, the difference between these two observed inclinations is evidently equal to the angle ACB where C is the centre of the earth. This quantity can, therefore, be observed.

It is also possible to obtain by measurement, the distance AB, *i.e.*, the distance between the two places A, B measured along the meridian through A, B.

And it is found from actual (geodetic) measurements, that the circular measure of the angle ACB varies as the distance AB along the meridian AB (very nearly), wherever the two stations are taken. This shows that the curvature[1] of every meridian is, very nearly, the same at every place. Hence, we conclude that the earth is, aproximately, a sphere.

18. The earth is an oblate spheroid.

Elaborate and very careful measurements have shown, that a meridian is an ellipse of small eccentricity, with its minor axis coinciding with the axis of rotation of the earth and that all meridians are practically equal. The shape of the earth is, thus, an oblate spheroid. [Fig[2] 12.]

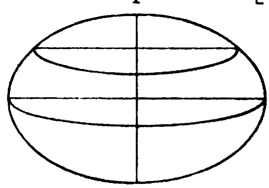


Fig. 12.

Major Axis = 3963·3 miles.

Minor Axis = 3949·8 „ nearly.

19. It is also known that a spherical body which is a viscous or a semi-fluid mass, if made to rotate about a diameter will assume the shape of an oblate spheroid, with the axis of rotation as the minor axis of its elliptic section.

[1] Curvature is obtained by dividing the circular measure of the angle between consecutive normals at the extremities of an elementary arc, by the length of that arc.

[2] Enormously exaggerated, of course.

The observed shape of the earth is, therefore, consistent with the hypothesis (which has been verified on independent evidence) that the earth was originally a semi-fluid mass.

20. If we take account of the eccentricity of a meridian of the earth, the vertical line at any place will be the direction of the normal to the meridian through the place. It is slightly inclined to the central radius to the place. This inclination is called the angle of the vertical [*e.g.*, $\angle$CPN, where C is the centre, PN being the normal or the vertical line at P (fig. 13)].

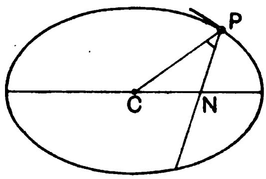


Fig. 13.

Obs. For most purposes, we shall regard the earth to be a sphere.

21. We have seen that to the observer, the celestial vault appears spherical in shape and that, therefore, if with the observer's eye, as centre, we describe a sphere, it will be similar to the celestial vault. Such a sphere is called the **celestial sphere** of the observer.

It is easily seen (fig. 14) that if lines drawn from the centre O of this sphere to the stars 1, 2, 3, meet the sphere at points $s_1$, $s_2$, $s_3$, then the directions of the stars will be given by $Os_1$, $Os_2$, $Os_3$, and the *relative angular* positions of the stars will be the same as those of the points $s_1$, $s_2$, $s_3$.

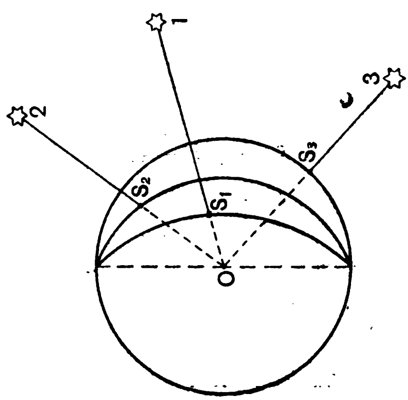


Fig. 14.

When the positions of all stars are thus marked on the celestial sphere, the whole of the celestial vault, *as it*

*appears to us at any time*, will have been completely mapped out.

Again, as all celestial distances, on account of their enormous magnitude appear to us to be equal (Ch. II, 2), any changes in the *linear* distances of celestial bodies from the observer are inappreciable also. Hence, the motions of celestial bodies, which we are cognisant of, ordinarily, are really changes in their angular positions only. Accordingly, these *motions* also can be adequately represented on the celestial sphere, as changes in the positions of $s_1$, $s_2$, $s_3$, etc., alone.

22. From what we have already stated, it will be evident that in the case of the vast majority of celestial bodies (*i.e.* stars and nebulæ), we have to be content mainly with a study of these angular changes only. Even in the case of the remaining few, the changes in their distances cannot be directly observed but have to be deduced by very careful observations and calculations. Accordingly, these changes will not concern us at present.

23. Now on a map of a portion of the earth's surface, it is necessary to fix the positions of certain cardinal lines and points before we can mark the positions of places on it. In the same way, in order that the celestial sphere may serve as a map of the celestial vault, at any time, we must begin by fixing the positions of certain cardinal points and lines on it.

24. Let ABP (fig. 15) represent the earth, C*p*, the axis, *p*, the north pole, AB, the equatorial diameter, *p*OB, a meridian through a given place, O.

Then, OP, parallel to C*p* will be the direction of the axis of the earth, as seen by the observer at O. In other words, if a star marked the direction of C*p*, that star, as seen by the observer at O would be in the direction OP, on account of the enormous distance of the star.

Also, if C is the centre of the earth, then CO will be the direction of the **vertical** (on the assumption that the earth is a sphere).

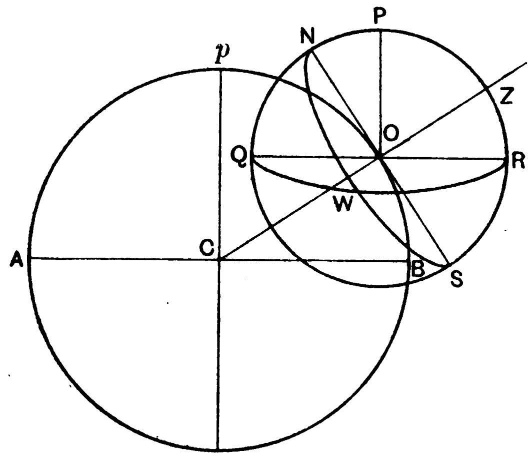



Fig. 15.

25. If now, a sphere is described with O as centre, this will be the celestial sphere of the place O.

The point at which CO, produced intersects the sphere will be the position of the zenith (Z) on the celestial sphere.

Or, the point vertically overhead on the celestial vault is called the **zenith** and the point on the celestial sphere which marks this point is the Zenith point (Z).

**Celestial pole.**—*Def.* The point at which the line OP drawn parallel to the earth's axis meets the celestial sphere is called the celestial pole (P).

It thus marks the direction of the *terrestrial* pole.

It is called the North pole, if the observer is in the Northern hemisphere.

**Horizontal plane.**—Remembering that the horizontal plane at O touches the earth at O, it is easy to describe it, for it is perpendicular to the line OZ.

**Celestial horizon.**—The intersection of this plane with the celestial sphere is necessarily a great circle. It is called the celestial horizon (NWS).

26. Again, the great circle in which the terrestrial meridian $pOB$ intersects the celestial sphere contains Z and P and the line NOS, the tangent to the meridian at O.

**Celestial meridian.**—This great circle NPZS is called the **celestial meridian.** It is a vertical circle, since all great circles (on the celestial sphere) containing OZ are vertical.

27. *The* **North** *and* **South** *points.*—If P is directed to the North pole of the earth, the points N and S, or the points of intersection of the celestial horizon and the celestial meridian are the North and South points (the former being directed towards the north pole).

To find these, we have, first, to determine the direction of OP (that is, that of the polar axis of the earth). When this has been determined, let a telescope or rod be pointed along OP and then sweep out the vertical plane through it, till the telescope or rod is horizontal; in this position it will point along NS.

The **East** and **West** points.—The points of the celestial horizon at the angular distance of 90° from N and S are the East and West points.

These are the cardinal points.

28. **Celestial equator.**—The plane through O parallel to the terrestrial equator or perpendicular to OP intersects the celestial sphere along a great circle (QWR) called the celestial equator.

Since the angle BCO (fig. 15) is the latitude of O, the angle NOP is equal to the latitude and the angle ZOP is equal to the co-latitude, (or the complement of latitude).

It can be easily shown [1] that the points of intersection of the celestial horizon and celestial equator are, each, 90° from N and S. These points W and E are, therefore, the West and East points, on the usual convention.[2] [Fig. 16].

29. We are, now, in a position to describe the celestial sphere of a place, whose latitude is known. Take a sphere, with its centre at O. Draw the radius OZ in the direction of the vertical or to represent the vertical line, and describe the great circle, perpendicular to it to represent the celestial horizon.

Describe also a great circle through Z (fig. 16) perpendicular to the celestial horizon to represent the celestial meridian. Draw OP in the plane of this meridian, such that the angle ZOP is equal to the co-latitude of the place. Then, P will be the celestial pole and the great circle perpendicular to OP, will be the celestial equator.

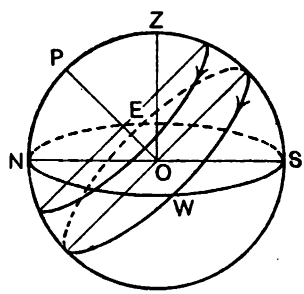



Fig. 16.

The intersections of the celestial meridian and the celestial horizon are the North and South points and the intersections of the horizon and the equator are the East and West points.

30. When these cardinal points and lines have been marked on the celestial sphere, the position and the motions of celestial bodies can be represented on it with comparative ease.

[1] Since P is the pole of QWR and Z that of NWP, W is the pole of NPZ or W is at an angular distance of 90° from N or S.

[2] If we imagine an observer to stand at O and look towards E, his left hand will point towards the North.

Thus, the (apparent) diurnal motion can be represented by means of small circles, parallel to the equator (with arrow heads to indicate motions from East to West).

31. When this has been done, we can classify celestial bodies into various groups, according to their paths on the celestial sphere.

(*i*) Bodies which move in fixed small circles with constant speed. These must be bodies fixed in space relatively to the earth, *viz.*, stars (and nebulæ).[1]

(*ii*) Bodies which move in small circles, the positions of which vary continuously.

32. In order to explain the motion of the second group, consider a point, moving along any path whatever on the celestial vault. Superpose on this motion, the diurnal motion of the celestial vault itself (which, as we know, is due to the motion of the earth, and therefore also of the observer, being opposite to this motion). The effect will be to give the point a motion in a spiral. The successive threads of this spiral will, however, be nearly circular, if the displacement of the point along its own path on the celestial vault, in one day, is inconsiderable and is, on that account, neglected. Thus, we conclude that the bodies belonging to the second group have motion on the spherical vault, *in addition to the apparent diurnal motion, i.e.*, a motion *among the stars*. The mode of representing this motion will be considered later.

33. It should be remembered, in this connection, that this motion (on the celestial vault) among the stars, itself may be real or apparent.

[1] The nebulæ being extended patches of light need hardly be considered in this connection.

34. Returning to the first group of art. 31, we observe that these can be classified into various sub-groups, *viz.*—

(*a*) Bodies that never set [as seen from the place of observation (fig. 17)]. These are stars whose diurnal paths lie entirely above the horizon.

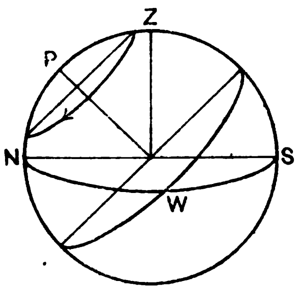


Fig. 17.

(*b*) Those that never rise, their diurnal paths lying entirely below the horizon (fig. 18). These two groups are called circumpolar stars.

(*c*) Those that both rise and set: Rise at points such as R, attain their maximum heights above the horizon, at their meridian passage (*e. g.* M) and then set (at T). [Fig. 18.]

*Def.* Such a star is said to culminate, when it is at the meridian.

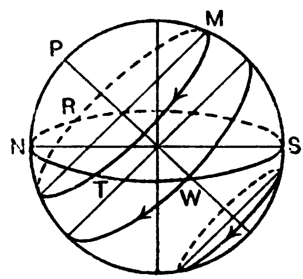


Fig. 18.

35. We, next, proceed to consider the mode of representing the position of a heavenly body on the celestial sphere. Referring to the general mode of representing a point on a sphere, we observe that we need only specify the two great circles to which the position must be referred. We have, accordingly, different modes of representation and corresponding systems of co-ordinates, by means of which the position may be defined.

*Def.* Co-ordinates are quantities, by means of which, the position of a point may be defined.

36. Let the great circles of reference be (1) the *celestial horizon* (I) and (2) *celestial meridian* (II) [Fig. 19].

If we describe the great circle ZSL (Ch. I, 13), then the position of a point S is defined by SL or the angle SOL, called the **altitude** and NOL[1] called the **azimuth**.

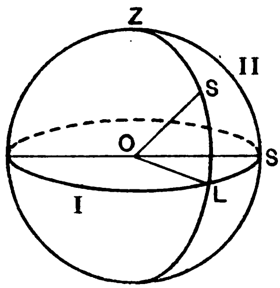


Fig. 19

*Defs.* Great circles perpendicular to the horizon are called VERTICAL CIRCLES. The vertical circle through a celestial body is called the VERTICAL of that body (*e.g.*, ZSL).

*Defs.* The ALTITUDE of a star is its angular height above the horizon or the angular distance of the star from the horizon (measured along the vertical of the star). Its complement is called the ZENITH DISTANCE.

THE AZIMUTH of a star is its angular distance from the North point measured along the horizon, in the direction of the apparent diurnal motion of celestial bodies.

37. Next, let the great circles of reference be (1) the *celestial equator* (I) and the *celestial meridian* (II) (fig. 20).

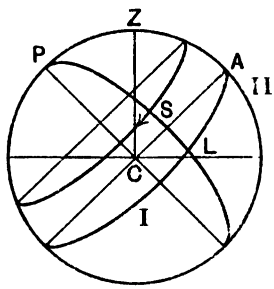


Fig. 20.

If we describe the great circle PSL to intersect I at rt. angles, the position of the star S is defined by SL or the angle SOL called the **declination** and the angle ACL or AL called the **hour angle**.

[1] The letter S on the great circle I should be N.

*Defs.* The great circle perpendicular to the celestial equator through any star is called the DECLINATION CIRCLE of the star (*e.g.* PSL, fig. 20).

The declination of a star is the angular height of the star above the celestial equator, *i.e.*, its angular distance from the celestial equator (measured along the declination circle of the star).

The hour angle of a star is its angular distance from the celestial meridian (measured along the celestial equator) towards the west point and expressed in hours, minutes, etc., calculated at the rate of 15° to the hour.

38. Obs. The zenith distance or the altitude of a star evidently depends on the place of observation, as it is measured from the horizon of the place, as well as on the time of observation, since it constantly changes, on account of the diurnal motion. In the same way, azimuth also depends on the place of observation, as it is measured with reference to the celestial meridian of the place as well as on the time, for it follows the diurnal motion of celestial bodies.

On the other hand, declination does not depend on the place of observation, as it is measured from the celestial equator, the direction of which is independent of the position of the observer. It is also independent of the diurnal motion and, therefore, also *of the time of observation*, in the case of bodies which are fixed in space and have, as such, only *apparent* diurnal motion.

But the hour angle depends on the place of observation, since it is measured from the meridian of the place and, of course, necessarily, also on the time of observation.

If, therefore, instead of this meridian, we take the declination circle through a fixed star or through a fixed[1]

[1] Fixed in the sense of being independent of the observer's position.

point in space, as the second circle of reference, the angular distance of a star from this latter circle will be independent of both the position of the observer and of diurnal motion. The fixed point chosen is the point of space which the sun occupies when he is on the equator. It is called the first point of aries. (Or, $\gamma$) [Fig. 21].

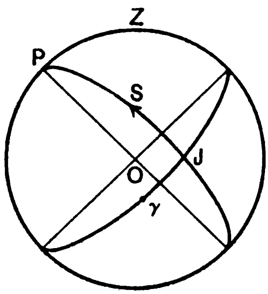



Fig. 21.

We have accordingly, the third system of co-ordinates in which the planes of reference are (*i*) the celestial equator, (*ii*) the declination circle through the first point of aries.

The co-ordinates in this case are JS, the declination and $\gamma$, the **Right Ascension** (written, for brevity, R. A.).

39. *Defs.* The **right ascension** of a star is the arc of the equator intercepted between the declination circle of the star and the first point of aries, expressed in hours, minutes and seconds, 15° of this arc being equivalent to one hour and measured in the *opposite* direction to that of the apparent diurnal motion of celestial bodies, *i. e.*, in the opposite direction to that, in which hour angle is measured.

40. Now, on account of the diurnal motion of the earth, the celestial vault appears to revolve about OP, with stars fixed on it. Hence, the R.As. of all stars must be constant. They change slightly, however, as the stars are not *absolutely* fixed in space.

Since the apparent diurnal motion of stars is entirely due to the rotation of the earth about its axis, the period of this rotation is equal to the interval between the successive passages of a star across the meridian of a place.

This period is called a **sidereal day**. Since $\gamma$ describes uniformly a whole circumference in one sidereal day, the sidereal interval (art. 39) that elapses since the meridian passage of a star is equal to the hour angle of $\gamma$. If, now, time is reckoned from the moment that $\gamma$ is on the meridian, and is called sidereal time, then the sidereal time at any moment is equal to the hour angle of $\gamma$.

41. If a star is at the meridian at any time, then the R.A. of the star is equal to the sidereal time of its meridian passage. For, evidently, the hour angle of $\gamma$=R.A. of the star, which is on the meridian at the moment.

This, as we shall see, enables us to determine the R.A. of a star. [Ch. IV, 13.]

42. Referring to the classification of celestial bodies that we had in art. 11, we observe that we may also classify them as follows.

(1) Bodies whose R.A. and declination are constant: These are the 'fixed' stars.

(2) Bodies whose R.A. and declination are continuously varying.

43. **Celestial globe.**—An astronomical or celestial globe is a chart of the entire celestial vault or a reproduction of a celestial sphere, on which the relative angular positions of celestial bodies are represented.

The globe is provided with an axis of rotation, made of brass which, passing through its centre, is fixed to a vertical circle of brass, surrounding it. This brass circle, in its turn, is capable of rotation in a groove cut in a horizontal circle of wood, also surrounding the globe and fixed to the stand on which it rests. When the globe is so placed, that it represents the celestial sphere of a given place, the operation is called, *rectifying the globe*. For this, the brass axis is made to point in the direction of the polar axis of the place of observation (roughly, in the direction

of the pole star). This ensures that the brass circle shall be roughly in the plane of the meridian and the points where it passes through the horizontal circle are the North and South points.

When the globe is thus placed, the upper portion of the globe is a chart of the celestial vault, as seen by an observer supposed to be at its centre. And the entire globe, represents the celestial sphere of the place of observation. If, now, the globe is rotated, we have a fair representation of the heavens, as seen by such an observer.

### EXERCISE.

1. Describe the appearances presented by the sky, when you stand on a broad flat plane, on a clear night. Briefly explain these.

2. Explain or justify the expression, celestial vault.

3. Define "the axis of the earth." You are provided with a large number of strings fixed to a point on the earth's surface, explain how with the help of these you will,

   (1) determine the axis of the earth,
   (2) describe the celestial sphere,
   (3) determine the cardinal points,
   (4) trace the diurnal path of a star,
   (5) measure the duration of a sidereal hour [with the help of (say, a sand glass), in addition].

4. Discuss Foucault's experimental method for proving the earth's rotation.

An observation is made with Foucault's pendulum, at a place in latitude 45°. Find the angle that would be turned through by the plane of oscillation in 20 sidereal hours. How long will it take to complete a revolution ?

5. Calculate the deviation of a particle dropped from the top of a tower (500 ft. high), at the equator ($g=32$).

6. Find the deviation of a particle dropped from the top of a tower of given height, at a given latitude. When is the deviation zero ?

7. Assuming the earth to be a sphere, find the distance travelled over by an observer from a place A to a place B, in the same meridian, if the zenith distance of a star changes by 10°, during the transit.

8. How are the cardinal points on the horizontal plane determined? Show that at all places, the celestial equator passes through the East and West points.

9. The declination of a star is zero. Prove that it always rises at the east point.

Show, by a diagram, the difference in the time of rising of two stars, whose declinations are given.

10. The R.A. of a star is 30°. Represent on a diagram, the position of the star at sidereal time 4 hours, if its declination is 60°, the latitude of the place of observation being 15°.

11. Explain by drawing a suitable spherical triangle, how the declination of a star can be determined, if the zenith distance and hour angle are 30° and 1 h. respectively.

Point out the difference that will arise in the nature of the problem, if the R.A. of the star is given as well as the sidereal time at the moment of observation.

12. If a star transits across the meridian to-day at a certain hour, when will it do so, a year hence?

13. What is the hour angle of the zenith?

An equaterial star is just rising at $0^h 0^m 0^s$. What is its right ascension, hour angle, zenith distance? Which of these quantities will change with time? Illustrate your meaning by means of a diagram, the place of observation being in latitude 70°.

# CHAPTER IV

## Astronomical Instruments

1. We have seen that in order to define the (angular) position of a celestial body, it is only necessary to define the direction OS, in which the star 1 is seen (fig. 14) and that this direction is completely determined by two angular co-ordinates.

2. For rough observations, if we look at the star through a tube of small bore and, then, rotate the tube about a horizontal axis (perpendicular to it) till it is horizontal, the angle it describes will measure the *altitude* of the star. While it remains horizontal, let it be rotated further about the vertical, till it points to the North; then the required angle of rotation gives the *azimuth* of the star. Such a tube was used at the *Manmandir* at Benares.

3. For accurate observations, however, we have to ensure (1) that the star is seen along a mathematically defined line and (2) that there are suitable mechanical contrivances by which the various rotations may be accurately effected, about proper axes and (3) that suitable means are provided for reading accurately the angles of rotation.

4. Now, when we look at a star through a telescope, we see it along a mathematically defined direction namely, the optic axis of the telescope. With a telescope, therefore, capable of rotation about suitable axes and provided with fixed graduated circles, perpendicular to these axes, we can determine the various co-ordinates, described in Chapter III.

5. Thus :—

(1) A Telescope capable of rotation about

(1) the Vertical and

(2) about different horizontal axes,

determines $\begin{cases} \text{Azimuth and} \\ \text{Altitude.} \end{cases}$

Such a telescope is called an **altazimuth**:

Let the telescope be pointed to a star. Then, if we rotate the telescope in the vertical plane in which the star is, till it is horizontal, we get the altitude. If, now, we further rotate the telescope till it points towards the North, we get the azimuth, the direction of rotation being opposite to the diurnal motion of celestial bodies.

(2) A telescope capable of rotation about the polar axis and about axes in the equatorial plane, similarly, determines

Declination and
Hour angle.

Such an instrument is called an **equatorial**. [Fig. 22.]

7. In practice, these angles are read off on graduated circles, whose planes are perpendicular to the axes of rotation. Thus, in the case of the Altazimuth, when the telescope points to the star, its inclination to the horizon is read off on a *vertical* graduated circle, carried with the telescope, while the inclination of the vertical of the star (Ch. III, 56), to the meridian is read off on a fixed horizontal circle.

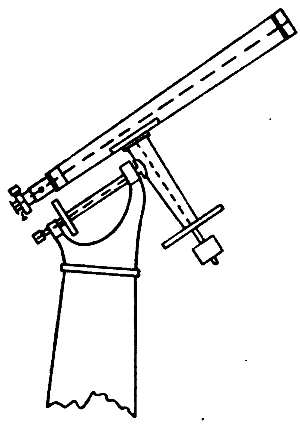


Fig. 22.

8. The mode of observation with the equatorial is similar. The telescope is pointed to the star and the

graduated circle which is carried with the instrument[1] is read off. Since the zero of graduation corresponds to the celestial equator (which is a plane perpendicular to the polar axis), the reading at once gives the declination.

At the same time, the inclination of the plane containing the optic axis of the telescope, and the polar axis, to the meridian plane is read off on a fixed graduated circle, perpendicular to the polar axis. This gives the hour angle (read in the direction of diurnal motion of celestial bodies from the north point).

9. It is clear, that in order to use a telescope as an equatorial, it has to be set, so that the fixed axis of rotation may point along the polar axis. This, as we have seen can be done, *roughly* by observing the pole star. Having set the axis in the direction of the pole star, we have to move it slightly, so that it may be in the meridian plane. For this, observe a circumpolar star and move the axis till the north polar distances of the star, at the upper and lower culmination are equal.

This will ensure that the axis does really point along the polar axis.

In practical working, the true direction can only be obtained by successive adjustments.

10. The larger instruments are provided with a clock-work arrangement, by means of which the telescope is made to rotate about the polar axis and describes a cone, with an angular velocity, equal to that of the earth. Thus, if the telescope is pointed to a star, it continues to be in the field of view, as long as it is above the horizon. In some instruments, there is a photographic attachment, by means of which photographs of stars may be taken.

11. In both these cases, the instruments have two modes of motion. If the telescope is heavy and it is necessary

[1] With its plane parallel to the optic axis of the telescope.

that it should be so, if it is to have considerable magnifying power, it is liable to get out of adjustment, on account of these various motions. In the case of the equatorial, moreover, since it has to be kept in an inclined position, this liability is all the greater. It results, accordingly, that observations with the aid of these instruments cannot give very accurate results. Nor is this necessary: The quantities that define the position of a star, independently of time and place of observation being (Ch. III, 38) its R.A., and declination, it is necessary to determine these and these alone with extreme accuracy. And from what we have just said, it will readily appear that the instrument which will determine these should have as few degrees of freedom as possible.

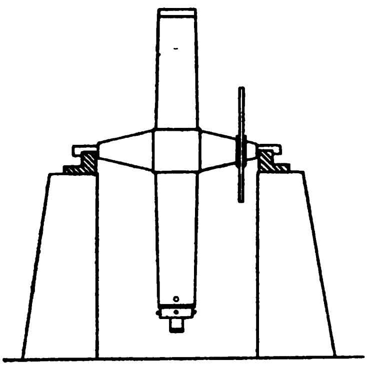


Fig. 23.

12. The instrument which determines these quantities with the greatest possible accuracy is called the **transit instrument** or, more properly, a **transit circle**. [Fig. 23.]

The transit instrument *is a telescope capable of rotation about a horizontal axis coinciding with the East and West line. It thus sweeps out the celestial meridian.* When it is provided with a graduated circle (art. 4) for the determination of declination, it is called a transit circle.

13. If the instrument is in ideal adjustment, when a star is seen through it, it must then be crossing the meridian. If, at the same time, we note the sidereal time, we at once get the R.A. of the star, for the R.A. of a star is the sidereal time of its meridine passage. [Ch. III, 41]

14. As to declination, we have already seen that it can be determined by means of the equatorial, but in view of the intrinsic defect of the instrument, the observed declination will not be sufficiently accurate. The determination of declination with the help of the transit instrument does not labour under this objection.

Let $\sigma$ be a star crossing the meridian and the figure 24 represent the celestial sphere of the observer. Also, let P, Z, A represent the same points as in figure 20 (Ch. III, 37), NS being the north-south line.

Then, since $AS = PZ =$ colatitude,

$A\sigma =$ the declination of the star (north),

$S\sigma =$ meridian altitude,

we get easily $A\sigma = S\sigma - AS$.

$=$ Meridian altitude—colatitude.[1]

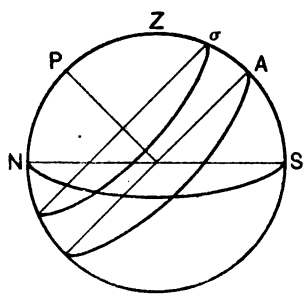



Fig. 24.

15. For the determination of declination, a transit instrument is provided with a graduated circle, fixed in the meridian plane, with its axis coincident with the axis of rotation of the instrument.

[1] For this, the zenith point should be accurately known. In order to determine the zenith point and the meridian altitude at the same time, the star may be observed directly, one night and its image after reflection at a trough of mercury, the following night Half the difference of the two readings gives the altitude and half the sum $\pm$ 90° gives the zenith point.

For accurate determination of R. A. and declination, it is necessary that the transit instrument with its graduated circle should be in ideal adjustment and time, accurately known (*i. e.*, the sidereal clock give correct time).

If this is not the case (and this can never be the case with absolute accuracy), the observed quantities will be affected with **errors**, which we now proceed to consider, in detail, in view of the importance, in Astronomy, of an accurate determination of R.A. and declination of celestial bodies.

16. In order that the transit instrument should be in ideal adjustment for the determination of R. A., it should satisfy the following conditions:

(1) The axis of rotation of the telescope should be horizontal;

(2) it should point due East and West,

(3) the optic axis of the telescope should be accurately perpendicular to the axis of rotation.

17. When (1) and (2) are satisfied, a line perpendicular to the axis of rotation sweeps out the meridian. When, therefore, (3) is also satisfied, the optic axis, *i. e.*, the line of sight through the telescope sweeps out the meridian plane. It is, therefore, only when all these conditions are satisfied that we can be certain, that a celestial body, when seen through the transit instrument (or the transit Circle) is accurately on the meridian.

18. If (1) is not secured, but both the other conditions are satisfied, the telescope will sweep out a plane intersecting the celestial meridian along the North and South line.

19. The apparent meridian in this case will be NZ′S (fig. 25), *i. e.*, a great circle slightly inclined (at $\angle$ZNZ′) to the true meridian. Accordingly, the *apparent* meridian passage of the star will occur, when the star is on NZ′S at

$\sigma'$ say, whereas the true meridian passage occurs when the star is at $\sigma$ ($\sigma\sigma'$ being the diurnal circle of the star).

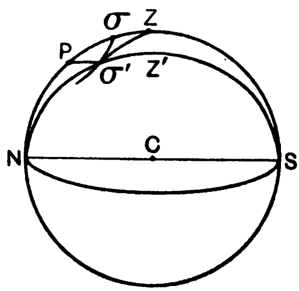



Fig. 25.

The difference in the hour angle corresponding to the two positions, $\sigma$ and $\sigma'$ measures the interval in sidereal units (Ch. III, 40) between the apparent and real meridian transits of the star and this is the error in the observed time of transit (positive or negative). The angle between the true and apparent meridian is called the *error of level.* [Art. 21.]

20. The adjustment of the level is made by means of a spirit level and levelling screws. It may, however, be that the adjustment cannot be completely secured by this means. It is necessary, in this case, to allow for outstanding error called *residual error.*

21. This can be detected by observing the upper and lower transits, across the meridian, of a circumpolar star. This interval ought to be equal to 12 sidereal hours. If the observed interval is less than this, the instrument must be in error and observation will give the actual error in time. For, from the figure 25, it is possible to calculate the relation between the level error and the error in the observed time of upper or lower culmination, (by means of the spherical triangle $\sigma'$PZ). Hence, when

the former is known from observation (art. 36), the latter can be determined.

22. If (2) is not satisfied, the telescope will sweep out a vertical plane (N′Z), intersecting the celestial meridian along the vertical (CZ). [Fig. 26.]

The apparent meridian will then be inclined to the real meridian at the angle NCN′. This is called "*Deviation error.*"

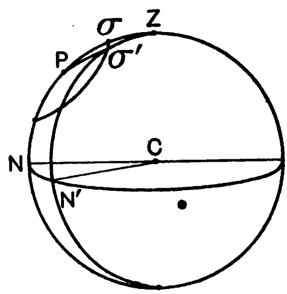


Fig. 26.

A star will appear to transit across the meridian at $\sigma'$, whereas the real transit occurs when it is at $\sigma$.

23. Accordingly, the interval $\sigma P \sigma'$ is the error in the observed time of transit, due to *deviation error.*

As before, it is possible to calculate the relation between the error in the time of transit and the angle between the apparent and the real meridian (by means of the spherical triangle $\sigma'PZ$) and thus, knowing from observation,[1] the instrumental error of deviation, the resulting error in time can be calculated.

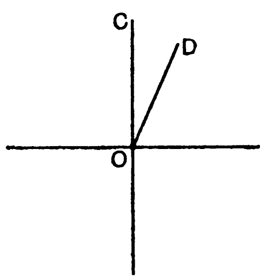


Fig. 27.

24. If the error (3) alone subsists, the apparent meridian will be a small circle, parallel to the actual meridian.

For let OD be the optic axis of the telescope and OC the mean line of the instrument, *i.e.*, the line perpendicular to the axis of rotation. [Fig. 27.]

[1] The discussion of the method is beyond our scope.

Then, in any position of the instrument, CO, OD, may be regarded as two lines rigidly connected together. Moreover, if O is the centre of the celestial sphere of the observer, having for its radius OD, D will be the position of a star, on the celestial sphere, when on the apparent meridian. [Fig. 27.]

As the telescope is rotated, therefore, D will describe a small circle, if OC describes the meridian on the supposition of the correct adjustment of the telescope as regards (1) and (2).

25. If we represent the effect of this error on the celestial sphere, we find that the apparent meridian will in this case be the small circle N′Z′S′, while the real meridian is NZS. [Fig. 28.]

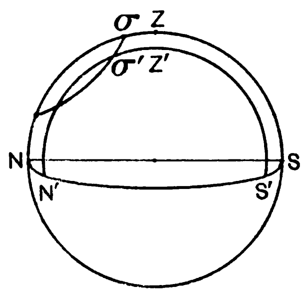



Fig. 28.

Thus, a star will appear to culminate at $\sigma'$, while it really does so at $\sigma$ and the error in the time of transit due to this defect in the instrument is the sidereal time corresponding to the path $\sigma\sigma'$. The inclination of the optic axis to the line OC (fig. 27) drawn perpendicular to the axis of rotation is called the *error of collimation* and, as before, it is easy to calculate the resulting error in the time of transit of the given star, when the error of collimation is known.

26. Now, when a telescope is turned towards the sky, it is found that a certain finite region, though somewhat limited in extent comes within view; this region is called the field of view of the telescope. And it is clear that a star will be visible through a telescope, as it enters the *field of view*.

27. Again, the rays from a source of light, say a star on passing through the object-glass of a telescope are brought to a focus, at a point which is called the image of the star. This image, moreover, lies in a plane called the focal plane of the object glass. And it is clear that when the image of the star coincides with the point of intersection of this plane with the line OC (fig. 27), it is accurately in the meridian (assuming that the errors have all been eliminated).

28. In order to fix this position, the telescope carries a reticle of five or seven vertical wires and (usually) two horizontal wires (fig. 29), so placed as to coincide with the focal plane of the object glass, while the point of the middle vertical wire, lying midway between the horizontal wires coincides (when the instrument is in ideal adjustment) with the image of the star at *its meridian passage.*

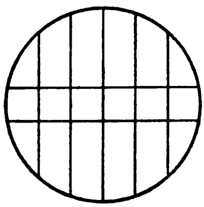


Fig. 29.

29. As the observer follows the motion of the star, after it enters the field of view, he notes the moments at which it passes across the various vertical wires and the mean of these times, taken according to a law, which can be determined (mathematically) gives the time of its meridian passage, with an accuracy, which could not have been attained, if there were only a single vertical wire.

30. *Def.* The line joining the optical centre of the object glass with the centre of the reticle (or the point of the middle vertical wire, lying midway between the two horizontal wires) is called the *line of collimation.*

31. In order to detect the collimation error, the telescope has to be pointed to a distant object, so situated

that it can be brought into coincidence with the centre of the reticle and then taken off the stand and reversed (without disturbing the stand). Then, the collimation error is nil, if the object still remains in coincidence with the centre. But if there is an apparent displacement, then the reticle has to be moved through half this displacement in the proper direction, in order to rectify the error. The transit is provided with adjusting screws for this purpose.

32. In order that the above method should be applicable, the telescope should be capable of being reversed. Provision is made in many instruments for this purpose: The axis of the telescope turns on accurately round, strictly similar pivots at its extremities, in Y-bearings, set on two fixed pillars and is so placed, that the whole instrument may be bodily removed and replaced.

33. Instead of a distant object, a *collimator* may be used. This is a horizontal telescope to the north or south of the transit, in the focus of the object glass of which a cross is placed, so that the cross can be viewed through the transit telescope.

As the rays from the cross issue parallel, after refraction through the object glass of the collimator, the cross serves the purpose of a distant object for all optical purposes.

34. Instead of one collimator, two similar collimators may be used, so placed, north and south of the transit, that the image of the cross of one of these coincides with the cross of the other. (In order to permit the light from one of the collimators to pass to the other, a cylindrical hole is left in the body of the transit, the axis of which coincides with the common axis of the collimators, when the transit is in the vertical position). When this is the case, the centres of the two crosses occupy diametrically opposite points, north and south.

35. When two such collimators are used, it is unnecessary to take the transit off its bearings. All that is necessary is to first look at the first cross, and then the other, and move the reticle so that the middle wire bisects the two images. The middle wire of the transit is then in the meridian plane.

36. A similar (optical) method may be employed for detecting the error of level:

For this, the telescope is pointed to a trough of mercury at the base of the instrument. If there is no error of level, the image of the middle vertical wire should, after reflection at the trough of mercury, coincide with the wire itself. If there is, this will not be the case. The displacement observed will be equal to twice the error.

37. In the working of the transit instrument, the cross wires are illuminated by light issuing from a lamp (suitably placed) which passes through an opening in one (or both) of the pivots and is then reflected on to the reticle by a mirror placed inside the body of the transit, inclined at 45° to the axis. The illuminated reticle is then focussed by means of the eye-piece. Thus, the object is focussed by means of the sliding tube which carries the reticle and the eye-piece, without disturbing their relative positions. Accordingly, the observer is able to note the position of the image in relation to the reticle, during the progress of the object across the field of view.

It should be noted finally, that the vertical wires should be tested for verticality, before an observation is begun. For this, while an object is in focus, the telescope is moved up and down slightly; and if the other adjustments have already been secured, such a movement will not produce any apparent displacement of the object, sideways. If there is such a displacement, the wires have

to be moved into accurately vertical positions by means of screws provided for the purpose.

38. Error and rate of the clock.—In determining the R.A. of a star or the sidereal time of its meridian passage, we must be able not only to assure ourselves as to the exact instant at which the star is in the meridian but also to tell correctly, what this time is. In other words, for a correct determination of the R.A. of a star, it is necessary that the clock time should be correct. But as no clock can be relied upon to give correct time always, it is necessary to know the *error* of the clock at any observation. For this, it is necessary to know the R.A. of a star, called a *standard* star *independently* of the clock. We shall see (Ch. VIII, 1) how this is done. Assuming, however, that such a standard star is available, we have only to note the time of its meridian passage. This ought to be equal to the *known* R.A. of the star. If this is not the case, the clock is in error and the known difference is the *error* of the clock.

39. If the clock time is noted at the *next* observation, and the corresponding error, then, if the clock is gaining or losing time, there will be a difference between the two errors and the average rate of the clock is known, for it is equal to this difference, divided by the interval between the two observations.

40. Assuming, now, that the rate is constant, we can find the error of the clock at any other time. But for accuracy, the error should be determined, as frequently as possible.

41. If the transit instrument is free from the errors enumerated above, it will sweep out the meridian plane and if, moreover, the clock is keeping correct time, the R.A. of a star can be determined with accuracy, provided we are able to note the exact instant, at which the image of the

star coincides with the middle wire. Any inaccuracy or uncertainty in this respect is almost wholly got rid of, by making observations with respect to the several wires.

42. For determination of declination, the reading circle should also be in ideal adjustment, in addition to the telescope. For this, the axis of the reading circle should accurately pass through its centre (the plain of the circle being vertical and parallel to the meridian), and, moreover, the graduation should be correct.

43. If the axis of the circle does not pass accurately through the centre, there will be an error, called the *error of centering*, which can be eliminated by taking two readings at opposite ends of the telescope.

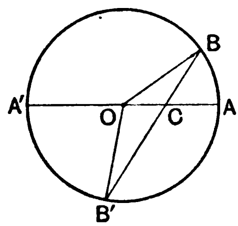



Fig. 31.

Let O (fig. 31) be the centre of the circle (with reference to which, the graduations are usually made).

Let OCA be the direction of the telescope at the initial position and B′CB, after rotation. Then, the amount of rotation is given by the angle ACB, while the apparent rotation, as given *by the graduation* is $\angle$ AOB, if the reading is taken at B alone.

But if, BB′ being the position of the telescope, readings are taken at both B and B′, *viz.*, the angles AOB and A′OB′ (where AA′ is a diameter) are read, then the mean of the two readings will give the required rotation.

For $\angle AOB + \angle OBC = \angle BCA$.

Also $\angle A'OB' - \angle OB'C = \angle A'CB' = \angle BCA$.

Therefore $\angle AOB' + \angle A'OB' = 2\angle BCA$.

44. In practice, there are six reading microscopes, symmetrically distributed round the circumference and keeping fixed positions. The mean of the results obtained with the three pairs of microscopes gives a result which eliminates not only the error of centering completely but also the error of readings of the microscopes, to a great extent, as well as the error of graduation—partially at any rate, if it is only slight.

45. A **Sextant** is a portable instrument for observing the angular distance between two bodies.

It consists of two mirrors, one of which is fixed and is called horizon glass, H (half of which is alone silvered, the other half being plain) and the other (moveable on an arm, called the index glass or index mirror) I. [Fig. 33.]

When H and I are parallel, a ray of light, after reflection at both mirrors will be received by the eye, in a direction parallel to its original direction. If now I is made to rotate through an angle $\theta$, the inclination of the final direction of a ray to the original direction[1] will be $2\theta$.

Hence, if an object O is seen by means of a ray which has suffered reflection at both mirrors and if another object O′ is seen at the same time directlly through H, the angle between O,O′ is equal to twice the angle between the two mirrors.

46. The mirrors are carried in a brass framework in the form of a sector of a circle, the arc of which is 60° and is graduated. One of the fixed arms carries the fixed mirror and the other, a small telescope (T). There is also a movable arm which carries the index glass. It should be further noted that the plane of the framework is perpendicular to the mirrors.

[1] It is easy to see that $\phi$ (fig. 33) = the angular distance between the objects O, O′ $= 2\theta$, where $\theta$ = angle between H and I, *i.e.*, the angle

47. The instrument is principally used for measuring the altitude of the sun. For this purpose, the direction O′H is taken horizontal. At sea, this is easily secured by directing the telescope to a point on the visible horizon, directly below the sun. In this case, a correction has to be applied for the dip.[1] On land, a shallow trough of

by which I is turned (H, being fixed), and HB is normal to mirror H and IB to I, while O′H is parallel to IB and to the optic axis of the telescope.

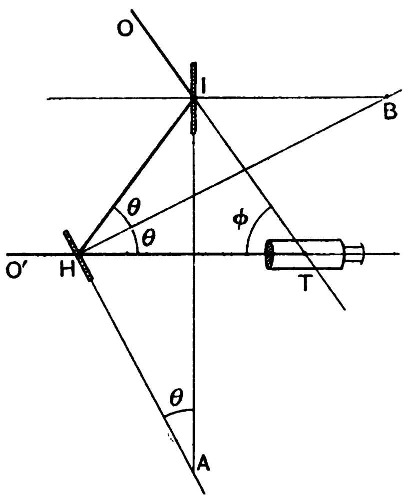



Fig. 33.

[1] If O is the observer, OT the tangent to the earth, supposed to be a sphere, then the inclination of OT to the horizon is called the dip (fig. 34)

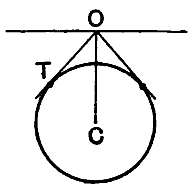



Fig. 34.

mercury represents the horizon (unaffected by dip), fairly accurately and the angle measured by the instrument is the angle between the sun and its image, reflected by the trough of mercury. The angle measured is, thus, twice the altitude.

48. It should be noted finally, that besides the instrumental errors, observations of celestial bodies are subject to other errors, due to the observer's position and motion, *viz.*, parallax, refraction, aberration, precession and nutation. In addition to these, observations are subject also to errors arising from defective power of observation and judgment of the observer himself. This is called personal error or personal equation.

## EXERCISE.

1. Describe an altazimuth. Explain how by means of a spirit level or otherwise, you could determine the zenith point.

A star is viewed directly through an altazimuth and again in the same vertical plane (say the meridian plane) after reflection at a trough of mercury. If the readings are 57° 13′ 15″ and 68° 26′ 45″, find the zenith point and the declination of the star.

2. The zenith distances of a circumpolar star at its upper and lower culminations are found to be 12° 7′ 15″ and 37° 16′ 25″ respectively. Find the latitude of the place and the declination of the star.

What instrument would you use, in making these determinations and why?

3. Find the declination of a star whose meridian zenith distance in a place whose latitude is 50° 22′ is 70° 10′ 10″.

4. The meridian passage of two stars differs by $10^h\ 15^m$ (sidereal). If the R.A. of one of them is $2^h\ 19^m$, find the R.A. of the other.

5. The zenith distances of a star, at upper and lower culminations are found to be 70° 10′ N. and 2° 15′ S. Find the latitude of the place and the declination of the star.

6. Two stars culminate at the same time and the angular distance between them is 10°. If the declination of one of them is twice that of the other, find these declinations.

7. Describe the transit instrument and explain how it is corrected for the various errors to which its readings are subject.

## CHAPTER V

### THE SUN

1. Of all the celestial bodies, with which we have to deal, the sun presents features, which easily distinguish it from the rest. Yet it is remarkable, though by no means strange, that only in comparatively recent times, have we deciphered the motion attributed to the sun, as due to the observer's own motion.

2. In order to trace the path of the sun, on the celestial vault, let us imagine ourselves as standing at the centre of a broad flat plane, from which the entire horizon could be seen. If we do so, we should see the sun rise *strictly* at the East point on the 21st of March and set, *strictly* at the West point on that day. If, however, we watched the sunrise and sunset the next day, or, more conveniently, a few days later, we should find the sunrise taking place at a point to the north of the East point and sunset at a point to the north of the West point—so that the line joining each of these two points would be nearly parallel to the East and West line. The line joining the points of sunrise and sunset moreover would move further and further north, till 21st June. After that, the line would go back towards the East and West line, till, on 23rd September, the sunrise would again take place at the East point and the sunset, at the West point. Thence, the line would move away to the South, till December 21st, after which it would move back to the East and West line again.[1]

[1] As a result of the above series of observations (which must have been continued by ancient observers for hundreds of years), it follows that ths sun does not rise in the East always but only on two days in the year. (It should be noted that the dates vary slightly.)

3. If we represent this on the celestial sphere of an observer in the northern latitudes, of which Z is the zenith, P, the north pole, NS, the horizon, and NWSE the celestial equator, then the positions of sunset on the 21st of March and 23rd of September will be indicated by W and those of sunrise at these epochs, by E. Similarly, on the 21st of June and 21st of December, the positions of sunset will be represented by $S_1$ and $S_3$ and those of sunrise by $S_2$, $S_4$, where the angular distances of the parallels through $S_1$ and $S_3$ are 23½°. [Fig. 35.]

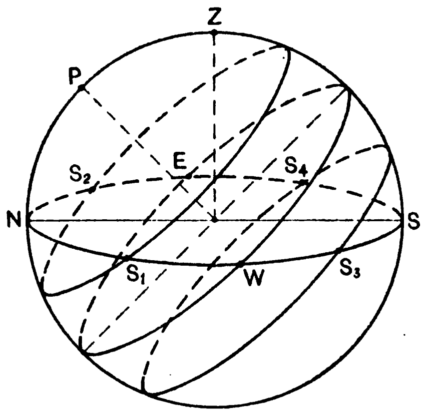



Fig. 35.

4. If the path of the sun on the celestial vault is observed, the most important conclusion to which we are led is that the path described through a year is a limited spiral (fig. 36) or if we neglect the tortuosity of the path during one day, a series of parallel circles, perpendicular to a certain line, fixed in direction in space which, as we now know, is the axis of rotation of the earth.

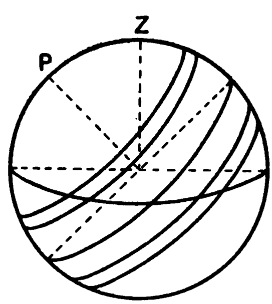



Fig. 36.

If, on the other hand, the path of a star is observed, it is found to be unchanged from day to day, being a fixed circle, in a plane perpendicular to the same axis.

We conclude, accordingly, that the motion of the sun, *as observed*, is a diurnal motion, common to all celestial bodies and an annual motion which belongs to it alone,—a motion relative to the stars, or *motion among the stars.*

5. In order to determine this motion, it is necessary (1) to determine the path relative to the stars, (2) the rate at which the path is described and (3) the time in which the entire path is passed over.

6. In order to find the path by observation, all that is necessary is to determine the position of the sun *relative* to that of a star at its meridian passage. But this direct method was not available in ancient times, on account of the fact that solar light necessarily shuts out of view all neighbouring stars. It was, accordingly, necessary to resort to indirect methods. One of these would be to observe, first, the motion of the moon, directly. When this has been done, it should be possible to trace the path of the sun, since the relative positions of the sun and the moon are known, especially at a solar and a lunar eclipse. It seems likely that a method of this kind was adopted by ancient astronomers, who discovered that the path of the sun passes through twelve groups of stars, distributed with some regularity, over an entire belt of the sky. These groups of stars are the so-called signs[1] of the Zodiac, the discovery of which

[1] The following are the names of the signs of the Zodiac (probably, from ζωογ, a living creature).

| | |
|---|---|
| Aries. | Libra, |
| Taurus, | Scorpio, |
| Gemini | Sagittarius, |
| Cancer, | Capricornus, |
| Leo, | Aquarius, |
| Virgo, | Pisces. |

The symbols used by the ancients are for the most part conventionalized pictures of the objects, named. The symbol for Aquarius is the

was undoubtedly the most remarkable achievement of ancient astronomy.

7. Before proceeding to discuss the methods adopted at the present day for determining the path of the sun, let us consider how this device was availed of, in ancient astronomy.

8. If we start with any point of time of reckoning, say from an equinox (or the date on which there is equal day and night throughout the earth), we find that each group of stars or a sign is passed over by the sun roughly in one-twelfth of a year: Hence, one method of describing the sun's (or the moon's) motion would be to name the sign and the position in that sign that the sun (or the moon) occupies, at any time. A division of the belt into 365 parts as in Chinese Astronomy or into 360 parts as is now generally accepted and was used in *Suryya Siddhanta* was meant to mark out the sun's daily motion and evidently made for increased accuracy. But in any case, the usefulness of the zodiacal system, under which the sun or the moon itself served to indicate the day or the month, in ages unprovided with accurate instruments of measurement cannot be overestimated.

9. When the paths (*among* the stars) of the planets Mercury, Venus, Mars, Jupiter and Saturn—which were alone known to ancient astronomers—were observed, they were found to be also contained within this same circular belt of the sky (art. 13) having an angular width of about 18°. 6/

From the point of view of the solar system, therefore, this belt of the sky presents a unique importance, to those

Egyptian character for water. The origin of many of the signs is not quite clear.

who had not as yet learnt to use the geometric and the instrumental methods of modern times. And it is noteworthy that in spite of this drawback, the observations embodied in ancient treatises seemed to have been conducted with considerable amount of accuracy.

10. On the modern method, the solar light is no drawback to observation, as the *intermediary* is the astronomical clock. This clock keeps sidereal time and is set, so that it indicates 0 h., 0 m., 0 s., when the first point of Aries is on the meridian. The angular distance of the first point of Aries from the meridian, when the sun is crossing it, is, therefore, indicated by it, for this distance is equal to the sidereal time of the sun's meridian passage, expressed in degrees at the rate of 15° to the hour. This is the right ascension of the sun or its distance from the first point of Aries, measured along the equator, while the position of the sun in the meridian circle, as it transits across the meridian determines its declination or angular distance from the equator.

11. In this way, the sun's position is known at each meridian passage and these various positions give its annual path *among the stars*. As observation is made always at the same meridian, the diurnal motion is completely eliminated.

12. It is evident that this method is alone suitable for accuracy, although it is possible to observe stars even in day time with the help of a telescope of suitable magnifying power. For with such, the diffused light of the sun may be weakened sufficiently to provide a dark background for the image of a star, while the brightness of the star remains unchanged. This, it should be noted, is due to the fact that even when viewed through the most powerful telescope, a star remains merely a point of light.

13. Thus, let A$\gamma$ (fig. 37) be the equator, P the pole, PSA, the declination circle of the sun at its meridian passage at any place and $\gamma$, the first point of Aries, on the celestial sphere of an observer, supposed to be placed at the centre of the earth. Then $\gamma$A is the R.A. of the sun and SA the declination as measured by the transit circle, $\gamma$ being the position of the sun when its declination is zero. [Fig. 38.]

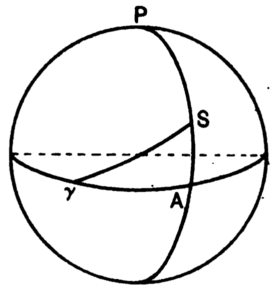


Fig. 37.

When the different positions of S have been thus marked, through a whole year, it is found that they lie on a great circle, inclined to the equator at $23\frac{1}{2}°$ and intersecting it at two points which are named the first point of Aries (marked, $\gamma$) and the first point of Libra ($\Omega$), the path being completed in a year (fig. 38). This path is called the **ecliptic**. It should be remembered that the point S not only moves along this path but is carried round, as seen by an observer on the earth, on account of the diurnal motion of the earth about the earth's axis. Thus, we get a limited spiral, which represents the sun's path on the celestial vault. [Ch. V, 4.]

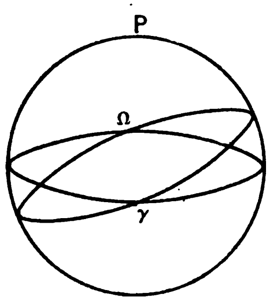


Fig. 38.

14. Having traced the path of the sun on the celestial vault, we have next to find its path in space. Here, it should be premised, that the (apparent) path of the sun *on the celestial vault*, as thus traced out need not be its *path in space*. For, on account of the magnitudes involved, as we

have seen (Ch. II), all celestial bodies *appear* to be at the same distance. Direct observation, therefore, is not competent to enable us to measure the variation in the distance of the sun from the observer. But since the angular diameter of the sun varies inversely as its distance, if we measure the angular diameter of the sun from day to day, and represent its reciprocal on a suitable scale, as the radius vector corresponding to any given celestial longitude (art. 16), we get the path of the sun in space.

15. Thus, let E (fig. 39) represent the centre of the earth, S the position of the sun, at any time, and S′, the point of intersection of ES, the line of sight with the celestial sphere of an observer at E:

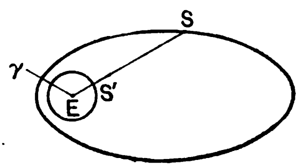



Fig. 39.

Then, the locus of S′ is the path of the sun on the celestial sphere or the celestial vault, but the locus of S is the path of the sun in space. The two, therefore, will not necessarily be similar. From the observed fact, however, that the locus of S′ (*i. e.*, the path of the sun on the celestial sphere) is a circle, we conclude that the actual path of the sun is a *plane curve.*

16. It is, moreover, clear that the locus of S′ is the ecliptic. If, then E$\gamma$ is the direction of the line of equinoxes, the angle $\gamma$ES′ is called the celestial longitude of S and can be determined.

17. To determine the orbit (*i.e*, the locus of S), we proceed as follows :

Observe the angular diameter of the sun at its meridian passage.[1] Then determine the corresponding celestial longitude, at the same time.

Since the diameter of the sun must be a constant length, the angular diameter must vary inversely as the distance from the earth. Now, take a length ES, inversely proportional to the observed angular diameter, and draw ES, inclined to E$\gamma$, at an angle, equal to the celestial longitude. The locus of S represents the path of the sun *in space*. [Fig. 39.]

18. Let $\gamma S'$ be the plane of the ecliptic on the plane of the paper and E the earth's centre. Then if $S'$ is the position of the sun on the ecliptic, it is, as we have seen, the intersection of the celestial sphere with the line joining the earth and the sun at any time.[2] Now, if $S_1$[3] represents its *actual* position at the time, the path traced out by $S_1$ will be known, if we know the relation between $ES_1$ and the angle $S'E\gamma$, since $E\gamma$ is a fixed direction in space.

19. Again, if $a$ is the circular measure of the angular diameter, corresponding to longitude $\gamma ES'$, then, we have, if R is the radius of the sun, and $S_1E$, the actual distance of the sun,

$$\sin a = \frac{R}{S_1E} \quad \text{or } a = \frac{R}{S_1E}, \text{ very nearly}$$

$$i.e., \ a. \ S_1E = \text{const.};$$

$$\text{but } a. \ ES = \text{const. [Art, 17]}$$

$$i.e. \ \frac{SE}{S_1E} = \text{const.}$$

[1] Roughly, by means of the sextant.

[2] Since the ecliptic is the curve of intersection of the plane of the sun's annual orbit with the celestial sphere of the observer, supposed to be at the centre of the earth. [Art. 13.]

[3] In S'S produced, not shown in the diagram.

Hence, the locus of S is similar and similarly situated to the actual path of the sun in space.

20. Assume that the path is an ellipse with E, as one of the focii. Then, if the angle $\gamma$ES$=\theta$, and the position of the major axis is known (art. 23), we should have

$\frac{l}{r}=1+e\cos(\theta-\beta)$, where $\beta$ is the longitude of the major axie or the apse-line, $l$, the latus-rectum of the ellipse and SE$=r$, $e$ being the eccentricity.

21. By taking two observations, we shall get two equations, from which $l$ and $e$ can be determined. When this has been done, any other pair of observations ought to give the same values of $l$ and $e$.

Since this is found to be the case, the original assumption that the path is an ellipse is justified.

In practice, the procedure to be adopted is much more complicated than this, owing to the fact, that our observations can never be made with absolute accuracy.

22. Although, theoretically, the above is the general method of determining $l$ and $e$, in practice, the method can be simplified.

From the property of the ellipse, if AA′ (fig. 40) is the major axis and C the centre, E being the focus, we have if $a$ is the length of the semi-major axis

$$EA=a(1-e),\ EA'=a(1+e).$$

where EA′ is inversely proportional to the minimum angular diameter (*i.e.*, on the 1st of July), and EA, to the maximum angular diameter (on the 31st December). Now these quantities are respectively, 31′ 31·0″ and 32′ 35·6″.

We have accordingly $\frac{1+e}{1-e}=\frac{32'\ 35{\cdot}6''}{31'\ 31''}$. Hence $e=\frac{1}{60}$ nearly.

Hence, $a = \dfrac{EA'}{1+e}$. Thus, $a$ varies inversely as the minimum (or maximum) angular semi-diameter.

In order to find the actual orbit, we must find the actual distance corresponding to *any one* observed angular semi-diameter, since the actual path is similar to the locus of S. [Art. 19.]

This is determined by the observation of solar parallax.

23. It has been found that both $e$ and $a$ undergo a slight variation. In order to obtain the orbit completely, it is not only necessary to find $e$ and $a$ but also the direction of the major axis or the apse-line. (This is assumed to be known in art. 20.) For when E, the focus, is known, we are able to describe the path completely, only if the direction and magnitude of the major axis and the eccentricity are known.[1]

24. In order to determine the direction of the apse-line, we proceed as follows :—

Measure the angular diameter of the sun, when the sun is at some point (S) of its orbit. Then, note the

[1] To show how to describe an ellipse, $a$, $e$, being known and the direction of major axis, as well as the focus.

Let E be the focus, EA, the direction of the major axis.

Cut off EA, EA′ (fig. 40) equal to $a(1-e)$, $a(1+e)$. Then AA′ will be the extremities of the major axis. Bisect AA′ at C. Then C will be the centre of the ellipse. Cut off CE′ = CE. Then E′ will be the other focus.

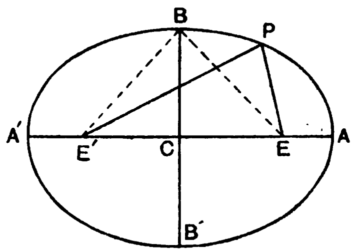


Fig. 40.

If, now, we attach the extremities of a thread of length $2a$, to two pins at E and E′ and keep the string stretched by means of a moving pencil, the pencil will trace out an ellipse. For, let P be the position of the pencil-head at any time, then $EP + E'P = 2a$. That is, the locus of P is an ellipse, having E and E′ for focii (maj. axis $= 2a$).

direction of ES′ where S′ is another point in its path, when ES′ is equal to ES, *i.e.*, when the sun's angular diameter is again equal to that at S. Now, since the apse-line bisects the angle SES′, by the property of the ellipse, the direction of the apse-line is determined. [Draw a diagram.]

25. It is, of course, difficult to note the exact position of the sun, when the radius vector drawn to it from E is exactly equal to ES. This difficulty is obviated, if we observe two positions $S_1'$ and $S_2'$ of the sun, where $ES_1'$ is a little greater and $ES_2'$, a little less than ES′. Then, the position of S′ can be accurately determined, on the assumption, that during the displacement of the sun from $S_1'$ to $S_2'$, the radius vector changes uniformly. This assumption will be justified, if the points are indefinitely near together.

*Note.*—The direction of the apse-line is not fixed. It is found to have a progressive motion (*i.e.*, motion in the direction of the sun's motion) of 11·25″, a year.

## Seasons.

26. The annual motion of the sun on the celestial sphere may be described as follows: (The dates are approximate).

On the 21st March, his declination as well as R.A. are both zero. From 21st March to 21st June, his northerly declination increases from 0 to 23½° and the R.A. also increases from 0 to 90°.

From 21st June to 23rd September, the northerly declination decreases from 23½° to 0°, while the R.A. increases from 90° to 180°.

From 23rd September to 21st December, the southerly declination increases from 0 to 23½°, while the R.A. increases from 180° to 270°. From 21st December to 21st March, the southerly declination decreases from 23½° to 0° and R.A. increases from 270° to 360°.

Near the epochs at which the declination is maximum, it changes very slowly. These are called **solstices** (lit. the epochs at which the sun is standing still).

27. It should also be noted that the sun is nearest to the earth on the 31st of December. He is, then, said to be in *perigee* (*i. e.*, nearest to the earth), while he is furthest from the Earth or is in *apogee*, on the 1st of July. The path of the sun, as it appears to the observer may, therefore, be represented, as in the annexed diagram (fig. 41), which shows (roughly) the relative positions of the line of equinoxes (art. 28), the line of solstices and the apse-line.

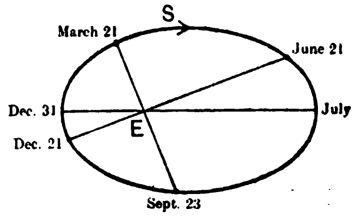



Fig. 41.

28. Now since on the 21st of March, the declination of the sun is zero, his diurnal path will coincide with the equator (WA) (fig. 42), assuming (Ch. III, 32) that during that day there is no change of declination. Accordingly, the period, during which the sun is above the horizon will be equal to the period during which he is below the horizon on that day. Thus, there will be *equal day*[1] *and night*, throughout the earth. The same will be the case on the 23rd of September. These are called **eqinoxes**. The line joining the positions of the sun at these epochs is the line of eqinoxoes. [Art. 27.]

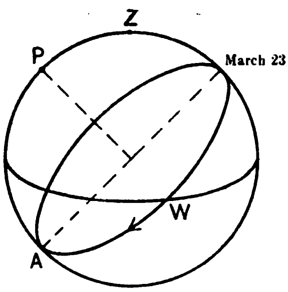



Fig. 42.

[1] Here 'day' means the duration of day-light.

29. From 21st of March to 21st of June, the sun's diurnal circles recede from the equator towards the North pole (P) [fig. 43]. Hence, for northern latitudes, between Lat. 0° and 66½°, the sun remains longer above the horizon than he is below it, during one solar day. Accordingly, during this period, the days are longer than the nights, the 22nd [or 21st] of June being the longest day (fig. 43). (CB is the diurnal path on 22nd June, where AC = 23½°.) The opposite is the case in Southern latitudes; that is, the days are shorter than the nights, from 21st of March to 21st of June. For the same reasons, from 21st June to 23rd September, also, the days are longer than the nights in Northern and shorter in the Southern latitudes.

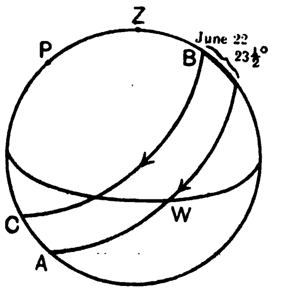



Fig. 43.

30. From 23rd September to 21st December, the diurnal circles recede from the equator away towards the south pole and accordingly, the sun remains longer below the horizon in Northern latitudes, than he is above it. [Fig. 44.]

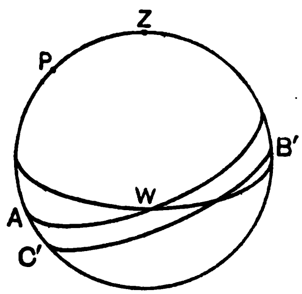



Fig. 44.

Hence, in these regions, the days are shorter than the nights, the 21st of December being the shortest day (C′B′ being the diurnal path on that day, where AC′ = 23½° and AW, the equator).

31. Finally, from 21st December to 21st of March, the days continue to be shorter than the nights, in Northern latitudes, the opposite being the case in Southern latitudes.

32. At a place in Lat. 66½° N, *i. e.*, a place whose co-latitude is equal to the obliquity of the ecliptic, there will be *one* diurnal circle of the sun (BC, fig. 45) which will be entirely above the horizon. For such a place, therefore, there will be one day (21st of June) of 24 solar hours, and one night (21st December) of an equal duration.

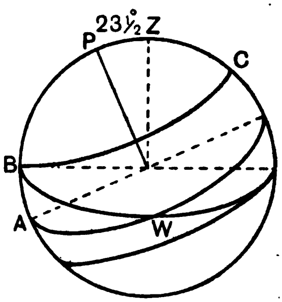



Fig. 45.

Similarly, at a place in Lat. 66½° S, there will be one day and one night, each, 24 hours long (21st December and 21st June).

33. For a place, whose Lat. is between 66½° and 90°, that is for places, within the arctic circle, it easily follows that from the moment the declination of the sun is equal to the co-latitude of the place, till it is again equal to this amount—during the whole of this period—the sun is entirely above the horizon (fig. 46), WS being the horizon AW, the equator and the angular distance of A from the north point, *less* than 23½°. Hence, if BC is the declination circle of the sun on the 21st June, BC will be entirely above the horizon.

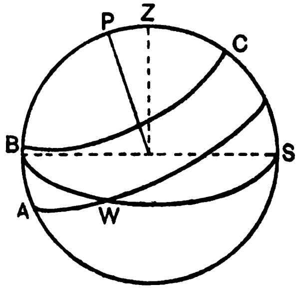



Fig. 46.

34. Thus, for all such places, the longest day as well as the longest night is more than 24 hours long.

35. Moreover, since near a solstice, the change of declination is slow, a small difference of latitude corresponds to a considerable change in the length of the longest day. Thus, the longest day increases from 24

hours to six months, as we pass from $66\frac{1}{2}°$ of latitude to the pole.[1]

36. It is this variation in the length of the day during the year that causes a variation in the seasons.

We have seen that in Northern latitudes from 21st of March to 21st of June, as well as from 21st of June to 23rd of September, the days are longer than the nights. Admitting that the accumulation of heat during the day bears some proportion to the duration of day-light and that the loss of heat during the night, similarly to the duration of night, we should conclude that there will be continuous accumulation of heat during this period and that the period from 21st June to 21st September will be hotter than the one from 22nd March to 21st June, for the latter is preceded by Winter, as we shall presently see, while the former is preceded by warm weather. Thus the

[1] The following table gives the actual results.

| Lat. | Length of the longest day |
|---|---|
| 0. | 12 hours. |
| 16 | 13 |
| 30·48 | 14 |
| 41·24 | 15 |
| 49·2 | 16 |
| 54·31 | 17 |
| 58·27 | 18 |
| 61·19 | 19 |
| 63·23 | 20 |
| 64·50 | 21 |
| 65·48 | 22 |
| 66·21 | 23 |
| 66·32 | 24 |
| 67·23 | 1 month. |
| 69·51 | 2 |
| 73·40 | 3 |
| 78·11 | 4 |
| 84·5 | 5 |
| 90 | 6 |

period from 21st June to 23rd September is **Summer** and the previous period, **Spring**.

37. Again from 23rd September to 21st December as well as from 21st December to 21st March (in Northern latitudes), the nights being longer than the days, there will be continuous loss of heat, so that these two periods will be colder than the rest of the year. Moreover, the second period will be colder than the first; for while the second period is preceded by one that is already cold, the previous period is preceded by hot weather. Accordingly, the period from 21st December to 21st March is **Winter** and that from 23rd September to 21st December is **Autumn.** Accordingly 21st March is called vernal equinox and 23rd September, autumnal equinox. Exactly opposite is the case in Southern latitudes. In these regions, for the same causes, the period from 21st March to 21st June is Autumn, and is followed by Winter, Spring and Summer. There is a corresponding variation in the obliquity of the rays received from the sun which operates in the same way (art. 38), so that, this subdivision is due to astronomical causes only. Local causes, however produce considerable difference in the actual climatic conditions of a place.

## Zones or Belts of the Earth.

38. At all places, of which the latitude is not greater than the maximum declination of the sun, he is overhead twice a year. This is the case when his declination is equal to the latitude of the place.

To these regions also, the solar rays never come very obliquely. Now, the amount of heat received per unit of surface varies as the cosine of the obliquity of the rays. Hence, the amount of heat received by all these regions of the earth included between two parallels whose angular distance from the equator is $23\frac{1}{2}^{\circ}$ N and S comprise the

hottest regions of the earth and are included in the **Torrid Zones**. On the other hand, the regions of the earth included between Lat. $66\frac{1}{2}°$ and $90°$ are the coldest regions, called the **Frigid Zones**. For in these regions, the longest nights are more than 24 hours long, while during the long days, there is comparatively small accession of heat, on account of the extreme obliquity of the rays.

39. In the regions of the earth between the Torrid and the Frigid zones, the extreme conditions of these regions do not obtain. They are called **temperate** zones.

40. It will, thus, be seen that in the Frigid zones, there will be practically one season, namely Winter, while at the equator itself, only one, *viz.*, Summer, or at most Summer and a rainy season.

41. The climatic conditions of the regions of the earth are also much dependent on various other causes, such as the distribution of land and water, Gulf streams, etc.

## CONSTITUTION OF THE SUN.

42. A complete description of all that the new methods have revealed and the ancient astronomers knew of the sun will lead us too far. We shall only confine ourselves to a brief description of the sun and the problems that he suggests.

43. The disc of the sun, as we generally see it, is the central and brightest portion. On account of the brightness of this central portion, the outlying portions, which are less bright, are not visible, except at a total eclipse of the sun. This central portion of the sun is called the *photosphere*. Surrounding the photosphere is a rather narrow region of coloured gases, chiefly hydrogen. This is called the *chromosphere* and surrounding the chromosphere is the *corona*, the crown or halo of glory. Protruding from the photosphere, are the so-called *prominences*

containing masses of vapour extending sometimes to thousands of miles. These can, as a rule, be seen at a total eclipse. But a method in use in several solar-physics observatories permits of their observation, daily. For this, the edge of the photosphere of the sun is focussed through a spectroscope,[1] so arranged that only light of one colour is allowed to pass through,—that of a prominence. In this way, the effect of the diffused light of the sun is eliminated.[2]

44. When the light of the sun is observed through a spectroscope, it is found to consist of seven coloured bands interspersed with dark lines, from which, as is well-known, the constitution of the atmosphere of the sun can be deciphered.[3]

[1] When a ray of white light is passed through a prism of glass (or any other transparent substance) it is not only deviated but is *dispersed*, that is, separated into rays of different colours (red, orange; yellow, green, blue, indigo, violet). This is called a SPECTRUM. In order to prevent overlapping of colours and consequent blurring, special arrangements are necessary :

Light is passed through a narrow slit which is placed at the principal focus of a lens, so that the rays from the source of light, on emergence through the lens are parallel. They, then, pass through the prism and the decomposed light is viewed through a telescope. Such an apparatus is called a SPECTROSCOPE.

[2] The bright lines of Hydrogen of which the prominences chiefly consist are separated from each other by the spectroscope into its different constituents but this act of separation does not weaken the intrinsic brightness of any of them. On the other hand the light from the edge of the sun produces a continuous spectrum, which can be spread over as long a space as one chooses. Hence, by using sufficient dispersive power, the bright lines of the flames can be made to stand out on the comparatively dark background of the Solar-spectrum.

[3] The following facts discovered with the help of the spectroscope are of importance in interpreting the meaning of the solar spectrum :

(1) White light (that of lime light or electric arc, for instance) produces a *continuous* spectrum, consisting of bands of all the seven colours.

45. When the photosphere of the sun is examined, it is seen—almost daily—to be interspersed with black patches, the so-called **sun-spots**. A very simple device for making their existence evident is to let sunlight pass through the object-glass of a telescope and fall on a white screen. The image on the screen shows these dark patches moving across the disc. If the screen is provided with a chart of the sun with its axis of rotation (for, as we shall see presently, the sun rotates about an axis) correctly directed, it is possible to note the positions and directions of motion of the various sun-spots observed, with reference to fixed lines (the equator and the axis) on the sun.

46. Another method which is also in use at several observatories is to photograph the sun at suitable times,

(2) Light from an incandescent *elementary* gas produces a spectrum which consist of bright lines, characteristic of the gas.

(3) When white light is passed through an incandescent elementary gas at a lower temperature, the continuous spectrum is seen to be interspersed with dark lines, which occupy the same relative positions, as the characteristic bright lines which the spectrum of the gas itself exhibits.

Now, when solar light is passed through a spectroscope, what really happens is that the white light of the photosphere loses, on passing through the atmosphere of the sun (which is evidently at a lower temperature), some of its constituents. This would indicate that this atmosphere contains vapours of substances which would yield spectra of bright lines, of which the white light of the sun has been deprived, in its passage through it.

In this way, it has been concluded that the following substances, among many others are certainly to be found in the sun :

Hydrogen, Iron, Calcium, Manganese, Nickel, Helium.

From the fact that stars yield spectra, similar to that of the sun, we conclude that they are self-luminous bodies like the sun. From a study of the dark lines they exhibit, stars may be grouped into classes, distinguished by the constituents of their atmospheres.

As Nebulæ give bright-line spectra, we easily conclude that they are masses of incandescent vapours. They consist mostly of hydrogen.

(in India, in early morning) when the various spots—if any exist—and there are always some—are seen as light patches on a dark background provided by the solar disc.

47. Appearance of sun-spots suggests that these are cavities in the sun, and that, when we are looking at a spot, we are really looking into the inner strata of the sun, which are conceivably at a lower temperature than the surface. Opinions differ as to the true origin of these spots. Faye conceives them to be the effect of solar storms. Secchi believes them to be dense clouds of eruption products, settling down into the photosphere, near to but not at the points where they were ejected. Whatever be their true nature, they have certain peculiarities which demand investigation; such for instance as their occurrence mainly in equatorial regions; their periodicity and their connection with magnetic storms on the earth. With regard to these, in the present state of science, we can only speculate. One evidence they afford is of far-reaching importance. The fact that sun-spots appear and disappear and that the time during which they are in sight is on an average roughly equal to the time during which they are out of sight, leads to the conclusion that they are carried round, while the sun *rotates* about its axis.[1]

48. We have already indicated the source from which, mainly, is derived the energy of the sun which is being

[1] The direction of this axis can also be determined from a study of the motion of these spots. It has been found that they appear to describe straight lines in June and December, while in September and March, their paths are most curved—with their convexity turned upwards or downwards, respectively, as seen, from the earth (from the northern latitudes). From this, we conclude that the solar axis of rotation is inclined towards the point, occupied by the earth in September. It has been further found that the motions of the sun-spots indicate that besides being carried round with the sun, they have also proper motions, depending on their position. There are other movements in the sun also, whose nature and cause are still under investigation.

dissipated away in radiation. Calculations based on this theory were made by Lord Kelvin but he did not take account of the possible nucleii of energy in the sun itself, such as radium. That such exist, there is now little doubt. But this means an unknown and practically inexhaustible source of energy. No sufficient data are, accordingly, available for estimating the past age of the sun or the future period of its existence, as a source of energy for the earth. Everything, however, points to the conclusion that the history of the sun and his system—for the life of the system is bound up with the sun—has had a beginning and will have an end. To quote Lord Kelvin:—"As probably there was a time, when the sun existed as matter diffused through infinite space, (art. 30, Introduction) the coming together of which has stored up its heat, so probably there will come a time, when the sun with all its planets, welded into one mass will roll a cold black ball through infinite space."[1]

EXERCISE.

1. What is the time of sun-rise and sun-set at any place, at the equinoxes?

2. Why is the sun never seen in the zenith in latitudes beyond 23½°?

3. What is the meridian altitude of the sun at a place, lat. 30. N. at the solstices and the equinoxes?

4. Find the latitude of the place, at which the meridian altitude of the sun at the summer solstice is 75° 21′.

5. The latitude of a place is 58° 27′ N. Find the meridian altitude of the sun at the place at mid-summer and mid-winter.

6. Find the average change of R.A. and declination of the sun, taken throughout the year.

[1] The whole course of Nature, in fact, points to a beginning and an end. And one begins directly to realize the inner meaning of the passage in the *Suryya Siddhanta*: "Then *Brahma* bearing the form of consciousness thought of creation."

If the change in the R.A. of the sun were uniform, find what would be its R.A., on the 1st of April, 22nd of June, 23rd of September and 1st of December.

7. Given the R.A. and the declination of the sun at a certain date, explain what geometrical construction, you would make, in order to determine the position of the ecliptic at that time.

8. If the inclination of the equator to the ecliptic were 90°, 0°, 29°, state what will be the effect on the seasons.

9. Comment on the fact that in Northern latitudes, the sun's distance from the earth on the 1st of July, *i.e.*, about mid-summer is greatest and is least on the 31st of December.

10. Represent on a diagram, the poles of the equator, the ecliptic and the horizon (P,Q,Z). Hence, show how the inclination of the ecliptic to the horizon changes, on the assumption that the horizon and the equator are fixed.

How would you describe this change—as real or apparent ?

11. How would you prove from observations that the (apparent) path of the sun round the earth is a plane curve ?

Explain how the changes in the angular co-ordinates of the sun, as seen from the earth are represented on the celestial sphere. Hence show how its path in space, relative to the earth is determined.

12. The angular diameter of the sun is observed at equinoxes and solstices. Would these observations be sufficient to enable you to determine its path in space ? Are they more than sufficient ?

Assuming the path to be an ellipse, write down equations which will embody the results of observation and hence deduce how far these are necessary or redundant.

13. Assuming the sun's orbit round the earth to be an ellipse, explain how you would determine the eccentricity and the major axis. Would these be sufficient to trace the path ?

14. What effect will the known motion of the apse-line have on the seasons, in course of time ?

Assuming that the rate is 1″, per year and the present inclination of the apse-line to the line of equinoxes is 20°, find when the summer and the spring will be of equal length.

15. Describe the constitution of the sun and its atmosphere. How do you conclude that the sun-spots are cavities in the sun ? What evidence is there of the sun's rotation ?

How are the constituents of the solar prominences analysed ?

# CHAPTER VI

## THE MOON

1. The next in importance to the sun is the moon, our nearest neighbour.

Even to the ordinary observer, the moon is the one celestial object that naturally enlists his interest and curiosity. Its constant changes of phase, its remarkable features and its rapid motion among the stars have made the moon necessarily the most interesting astronomical object, both before and after the invention of the telescope.

2. We have already seen that like the sun, the moon has a motion among the stars. The groups of stars through which the moon passes on the celestial vault, in completing its cycle round the earth were carefully studied and named by ancient Hindu Astronomers. These are the lunar asterisms or *mansions*, which enabled the position of the moon to be indicated at any time and is of an older date than the solar Zodiac. As, however, the groups of stars, through which the lunar path lies are not spaced out with anything like mathematical precision, the system could never, at any time, have lent itself to the purposes of an exact statement. Modern Astronomical methods alone could completely solve the problem of lunar motion.

3. In order to determine the path of the moon, we have to resort to the same method, as in the case of the sun. We determine, in the first place, the R.A. and declination of the moon's centre at its meridian passage at any place and thus obtain its path on the celestial sphere

of the observer (Ch. V, 10). If these quantities are corrected for parallax (Ch. X), the path obtained will be that with reference to the celestial sphere of the observer, supposed to be at the centre of the earth.

4. When this is done, it is found that the path of the moon describes, on the celestial sphere, a great circle inclined to the ecliptic at 5° 9′.

5. The path on the celestial vault, however, as seen by an observer on the surface of the earth, is, on account of the earth's diurnal motion, a limited spiral (Ch. V, 4), traced on a sphere.

6. Comparing, therefore, the motion of the sun with that of the moon among the stars, we observe that both the sun and the moon appear to have a motion round the earth.

7. Now, it can be proved that the annual motion of the sun round the earth is only apparent motion—that it is the earth, that is in motion and that the sun is, in reality, at rest. This being admitted, it necessarily follows that the motion of the moon cannot be also apparent motion. In other words, the moon must be *actually* moving round the earth. For, obviously, the earth cannot be moving, at the same time, about two bodies in two different orbits in two different planes. Thus, the truth of the *heliocentric* view of the solar system (Ch. VII, 10), necessarily involves the *geocentric* view of the moon's motion.

8. We have, next, to determine the actual **path** described by the moon, the projection of which on the celestial sphere is, as we have just seen, a great circle, slightly inclined to the ecliptic.

9. For this, we proceed exactly in the same way (Ch. V, 17), as in the case of the sun. Since the path on the celestial sphere has been already determined, we know the *direction* of the line of sight, *i.e.*, the line joining the moon

and the centre of the earth (provided we use the path on the celestial sphere of the observer, supposed to be at the centre of the earth), at any position and the corresponding angular diameter of the moon gives the distance on a suitable scale.

10. This gives the nature of the path. And it is found to be an ellipse with the earth's centre at one of the focii, the eccentricity of the ellipse being (on an average) equal to ·055 nearly.

11. If, finally, the actual distance of the moon from the earth *in any* one *position* is determined, the dimensions of the orbit are completely known, the greatest and least distances being respectively 252,900 miles and 221,600 miles, nearly. (Mean distance = 23,800.) And it is, more- 8
over, found that the moon's orbital motion is subject to the three laws of Kepler (Ch. VII, 7).

12. The motion of the three bodies (the sun, the moon and the earth) may accordingly be thus described:

The sun is a fixed star, rotating about an axis. The earth, also, rotates about an axis, and describes, moreover, an elliptic path, with the sun at one of the focii. Finally, the moon also rotates about an axis (art. 18), while describing an elliptic orbit with the earth at one of the focii, in a plane inclined to the earth's path about the sun, at an angle of 5° nearly.[1] The directions of rotation and motion are related in both cases, in the same way as that of a sphere on a perfectly rough surface, the motion of both presenting to the sun, the same aspect.

13. It follows therefore, that the period in which the moon describes its orbit about the earth, as seen by an observer on the earth is different from what this period

[1] The orbital motion of the moon in space, *i.e.*, with reference to the sun would, therefore, be of a complicated nature and might at first sight appear to be as complex as that of the planets. It is actually found

would have been, if the earth were at rest. The former period, *i.e.*, the interval from conjunction (Ch. VII, 16) to conjunction is called the synodic period, while the latter is called the sidereal period.

*Def.* **Synodic period** is the interval between two successive conjunctions (of the same kind).

**Sidereal period** is the time of passage of a body round the sun from one star to the same star again as seen from the sun as a fixed body.

In other words, the synodic period is the period round the sun *relative* to the earth, while the sidereal period is the *absolute* period round the sun. In the case of the moon, the former period is the interval from one new (or full) moon to the next, or one **lunation**.

however, to be much more simple, being an oval curve, which is always, concave towards the sun.

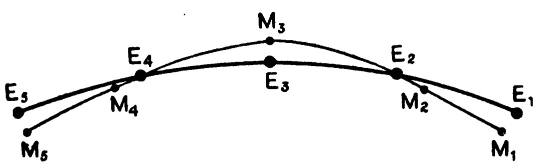



Fig. 48.—Moon's Path with Reference to the Sun.
$M_1$ and $M_5$, New Moon; $M_2$, First quarter.
$M_3$, Full Moon; $M_4$, Third quarter (Art. 18); $E$, $E_2$, the orbit of the earth.

*Cf.* The geocentric orbit of Jupiter as given by Cassini.

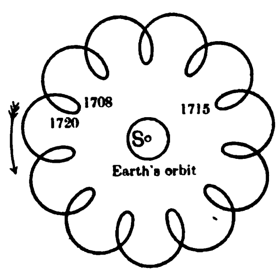



Fig. 49.
Geocentric orbit of Jupiter from 1708 to 1720.

Now, to find the relation between the two periods, if $\omega_e$ = the angular velocity of the earth round the sun,
$\omega_p$ = the angular velocity of the moon round the sun,
then $\omega_p - \omega_e$ = the relative angular velocity.

Again, if P is the sidereal period of the moon and E. that of the earth (E, being thus, the sidereal year), then, by the definition of angular velocity,

$$\omega_e E = 2\pi = \omega_p P.$$

Similarly, if S is the synodic period of the moon

$$S(\omega_p - \omega_e) = 2\pi.$$

This gives $\frac{2\pi}{S} = \frac{2\pi}{P} - \frac{2\pi}{E}$, which is the relation between moon's synodic and sidereal periods. When the synodic period has been determined by observation, the sidereal period can be found by calculation, with the help of the formula :

$$\frac{1}{P} = \frac{1}{S} + \frac{1}{E}.$$

$$\begin{aligned} \text{Now } S &= 29{\cdot}53059 \\ E &= 365{\cdot}25635 \\ & \quad d. \; h. \; m. \; s. \\ \therefore \quad P &= 27 \;\; 7 \;\; 43 \;\; 11 \end{aligned}$$

$$\text{or, approximately, } P = 29{\cdot}5 \left[1 - \frac{29{\cdot}5}{365{\cdot}25}\right]$$

$$= 29{\cdot}5 - 2{\cdot}3 = 27{\cdot}2 \text{ days.}$$

14. The points of intersection of the moon's orbit with the ecliptic are called the moon's nodes. Accordingly, the line of intersection of the plane of the lunar orbit with that of the ecliptic is called its line of nodes.

This line is not fixed in space but has a *retrograde* motion along the ecliptic of about 19°, each year. It, thus, completes a cycle in about 18⅔ years. [*Cf.* precession.]

This is the *sidereal* period of the revolution of the moon's nodes. The synodic period, that is, the period relative to the observer on the earth can be calculated by the formula,

$$\frac{1}{S}=\frac{1}{T}-\frac{1}{E}. \quad [\text{Art. 13.}]$$

where T is the sidereal period.

And remembering that the motion is *retrograde*, we get

$$\frac{1}{S}=\frac{1}{18\frac{1}{2}}+\frac{1}{1},$$

*i.e.*, the synodic period $=\frac{18\frac{1}{2}}{19\frac{1}{2}}$ of a year

$$\doteq 365\tfrac{1}{4}\times\frac{37}{39}=346{\cdot}62 \text{ days.}$$

(without regard to sign).

*Obs.* As in the case of the earth's path, the apse-line of the lunar orbit round the earth has also a *progressive* motion which is, however, much more rapid, being nearly 40° in a year.

15. The **phases** of the moon.

Like the planets (Ch. VII, 18), the moon undergoes changes in appearance, called its **phases**, which are, for obvious reasons, much more striking than those of the planets. The explanation is simple: The moon being an opaque body, that half of it which is turned towards the sun is illuminated. And, of course, the portion of the illuminated surface, turned towards the earth measures its phase. And it, evidently, follows that the relative positions of the sun, the moon and the earth determine these phases.

Let E, M (fig. 50) be the centres of the earth and the moon and ES, MS, lines from E and M, directed to the

sun. Then, if the plane of the paper represents the plane containing EM and MS, ACBD is the section of the moon by this plane, where AB is perpendicular to MS and CD, to EM.

16. Now, the plane perpendicular to MS (and containing AB) separates the illuminated surface of the moon from the dark portion, while the plane perpendicular to CD separates the visible from the invisible portion, as seen from earth. Hence, the *lune,* of which BD is the trace is the only portion of the illuminated surface, presented to the earth. Moreover, the visible portion of the moon, though, in reality, spherical, appears as a disc which is the projection of this spherical surface on the plane perpendicular to EM, the line of sight. When the lune is a quadrant, the projection is half-moon and when the lune is greater than a quadrant, the corresponding phase is said to be *gibbous.*

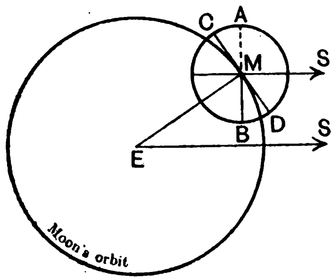



Fig. 50.

17. In the Figure 51, the plane of the paper is the plane, perpendicular to the line of sight EM of Fig. 50 and GH, the perpendicular to the plane CBD, while GBHD is the illuminated portion, as presented to the observer. [The letters are the same in both figures.]

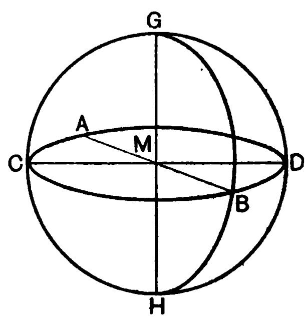



Fig. 51.

Now, since the projection of the lune GBHD on the disc, presented to the observer is the *crescent* seen by the observer, the breadth of the crescent is evidently $= r(1 - \cos\theta)$, where $\theta = \angle$ BMD and $r$ is the radius of the disc.

But $\theta = 180° -$ angle EMS

$=$ the exterior angle subtended at M by ES. [Fig. 50]

$\therefore$ the phase varies as $r\ (1 + \cos.$ EMS).

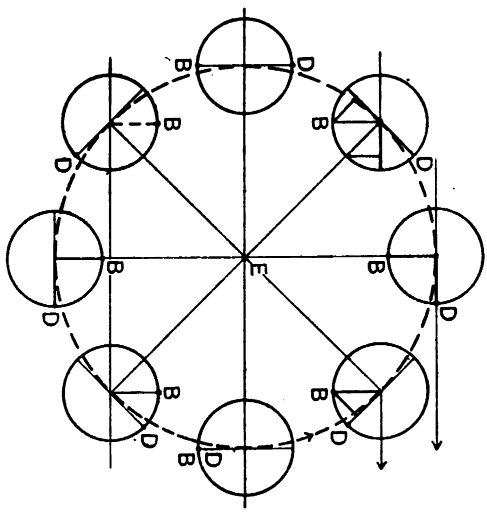



Fig. 52.

Hence, we conclude that

when EMS $= 180°$, *i.e.*, M is between the earth and the sun, the phase is zero. This is called the **New Moon**.

When the angle EMS $= 90°$ (*i.e.*, at first and third **quarters**), the phase is half (and the breadth of the illuminated portion $= r$). Finally, when the angle EMS $= 0$, the phase is full. This is **Full Moon**.

In the accompanying diagrams (Figs. 52 and 53), these various relative positions and the corresponding phases are represented, MS being always parallel to ES. [Fig. 50.]

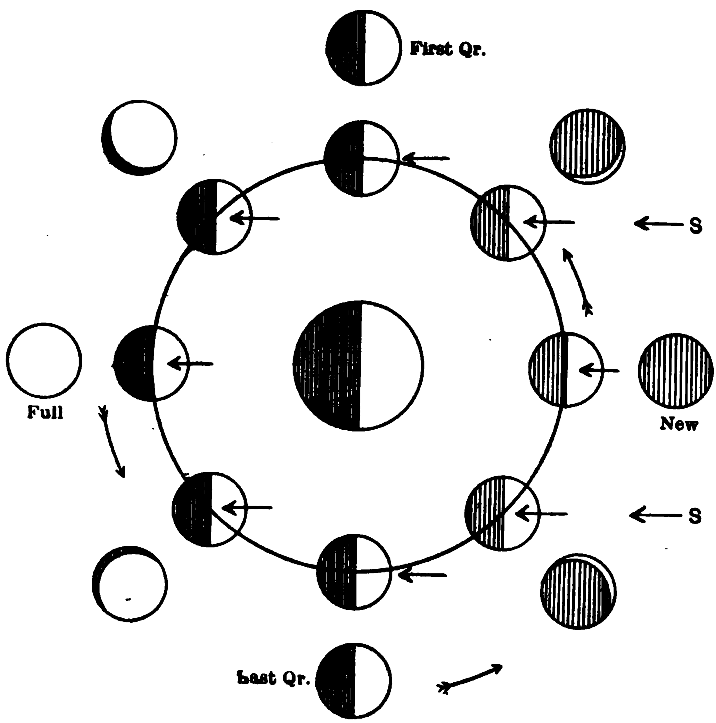



Fig. 53.

Phases of the Moon.

18. **Rotation.**—When the moon is carefully observed, it is found that certain markings on it occupy *nearly* always the same position, relative to the disc, presented to the observer. This necessarily leads to the following conclusions :

(1) That the moon rotates about an axis, *nearly* perpendicular to the plane of its orbit, round the earth,

and (2) the period of rotation about the axis is *nearly* equal to the period of its revolution about the earth.

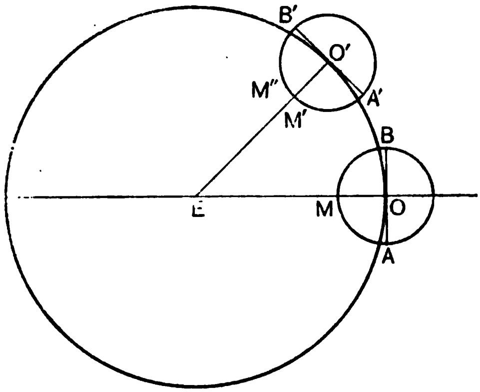



Fig. 54.

For let AMB (fig. 54) be the trace on the plane of the paper of the lunar hemisphere and let OO′ be its orbit, also on the plane of the paper. Let us suppose, also, that the orbit is circular, and M, the position of a certain mark on the moon—say in the middle of the arc AMB (centre O).

Let, now, A′M′B′ be the trace of the hemisphere, presented to the earth, in the second position (centre O′), M′ being the middle of the arc A′B′.

Now, suppose M′ is the same mark, which occupied the position M previously ; then, it necessarily follows that the hemisphere (AMB) of the moon is the same as A′M′B′ in the second position.

Now, the rotation of the moon in the interval is given by the inclination of the fixed line A′B′ of the body with a fixed line in space, *viz.*, AB. This rotation is therefore, evidently, equal to the angular displacement of the moon in its orbit. If this relation always held, it would follow that (1) the period of rotation is equal to the period of revolution and (2) the axis of rotation is perpendicular to the orbital plane.

19. It is, however, found that M does not occupy exactly the position M′.

Let us suppose that it comes to the position M″, in the arc A′M′B′ just a little to either side of M′. In this case, the angular displacement of the moon from O to O′ in its orbit will not be equal to the corresponding angular rotation.

In fact, since the rate of the angular displacement of the moon in its orbit obeys Kepler's first law and is, therefore, not constant, the amount of rotation in any given interval would not necessarily be equal to its angular displacement, especially, if the angular velocity about its axis is constant, as is found to be the case. This is also verified by observation, for it is, in fact, found that these quantities are not always equal. It will follow, therefore, that the plane perpendicular to the line of sight is not absolutely fixed with reference to the moon. In other words, sometimes, a small portion around the Western and sometimes a small portion around the Eastern edge, beyond the hemisphere limited by AB will be visible. This is called **Libration in Longitude**.

20. Let us next suppose M to come to a position $M_2$, lying on a line perpendicular to the arc A′M′B′ very near M′. This will be the case, if the axis of rotation is not exactly perpendicular to the orbital plane. It is known to be actually inclined at a constant angle of nearly $88\frac{1}{2}^\circ$ to the ecliptic. It results, accordingly, that the plane perpendicular to the line of sight does not always contain this axis; whence, it follows that sometimes, a little more of the Northern and at other times, a little more of the Southern region will be visible, than is, ordinarily, the case. This is called **Libration in Longitude**. Both these librations evidenced by the displacements of marks, say M, in the neighbourhood of M (which may be considered to be made up of displacements MM″ and $MM_2$) have been observed.

21. Again, the portion of the moon presented to the observer is included within the tangent cone, drawn from the observer to the moon. The lunar disc presented to the observer does not, therefore, accurately pass through the centre of the moon (though it very nearly does so, on account of the smallness of the angle of the cone), as we have assumed, in our discussions, so far. It is also clear that, as the observer is carried with the earth, in its diurnal motion, this cone will slightly vary its position in relation to the moon, so that the portion presented to the observer varies slightly during the day. This is called **diurnal libration.**

22. When all these librations are taken together, we find that, as a matter of fact, we have on the whole, brought within our purview, more than half of the moon, about 41 p. c. being never invisible and about 41 p. c. never visible.

23. Since the average change of R.A. of the sun is about $1^\circ$ per day and that of the moon, about $13^\circ$ per day (Art. 3), the moon gains upon the sun on an average $12^\circ$ per day (actually $12^\circ\ 11'\ 4''$). Hence, remembering that the R.A. is measured in a direction, opposite to the direction of apparent diurnal motion of celestial bodies, we conclude, that the moon's change of hour angle by $360^\circ$ bears the same ratio to the sun's change of hour angle by $347^\circ\ 48'\ 56''$ $(360^\circ - 12^\circ\ 11'\ 4'')$ as the average interval between successive transits of the moon bears to 24 hours. Thus, the average interval between the successive transits of the moon is equal to 24 h. 50 m. 36 s.

24. There is, thus, an average daily *retardation* of the moon, of about 51 minutes in its time of meridian passage (and that of rising and setting). That is, the moon rises nearly 51 minutes later, on an average, every day.

The Diagram 53 clearly brings this out also, which, indeed, is a familiar experience. For, while the full moon rises practically at sunset, the moon-rise lags behind, the next day and the lag continues to increase.

On account, however, of the irregularity in the moon's motion in R.A., the retardation in the meridian passage is subject to considerable variations. Both on account of this and also on account of the fact that the moon changes its declination, considerably (from 0 to 28° 36′), the daily retardation in the time of rising and setting varies considerably throughout the year.

The greatest interest attaches to the epochs at which the retardation is greatest and least.

## Harvest Moon.

25. In order to simplify the problem, we shall assume the moon to move along the ecliptic.

Let SN (fig. 55) be the horizon, EQ, the celestial equator, $\gamma$M, the position of the ecliptic, at any time and M, the position of the moon at the horizon, *i.e.*, when rising on a certain day. Also—in order to fix our ideas—let us suppose that the sun is setting at the same time.

Let MM′ be the average relative displacement of the moon in one solar day. Then, M′ is the position of the moon at the next sunset, and the moon-rise occurs, only when it is carried to the horizon on account of its diurnal motion—to R, where M′R is a portion of its diurnal path.

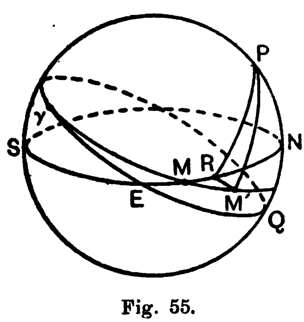



Fig. 55.

Hence, the time corresponding to the hour angle M′PR measures the retardation in the time of moon-rise,

*i.e.*, the interval between sunset and moon-rise, the next day.

Now, since, MM′R is a small spherical triangle, we may take it to be a plane triangle.

And, further, since MM′=relative displacement of the moon along the ecliptic in a solar day, this may be taken to be constant.

Hence, it is easy to see that the hour angle RPM′ will be least if (1) the moon's declination is least (*i.e.*, zero)[1] and (2) the length RM′ is least. Now RM′ will be least, if the angle M′MR (that is, the inclination of the ecliptic to the horizon) is least, since the declination being zero, the angle MRM′ may be regarded as constant.

In order to find when this inclination (M′MR) is least, we proceed as follows. Let K be the pole of the ecliptic, and P, the North celestial pole; then, KP represents the obliquity of the ecliptic. Therefore, as the ecliptic moves on the celestial sphere (relatively to an observer on the earth), K will describe a small circle about P. Now the length of the arc of the great circle passing through the zenith, Z and K measures the inclination of the ecliptic to the horizon. And this is evidently least, when K is on the celestial meridian of the observer *and lies between* P *and Z*—that is, when the ecliptic passes through the intersection of and lies between the horizon and the equator. For this, it is necessary that $\gamma$ should be at the East point, for northern latitudes, since the ecliptic passes to the North of the equator at $\gamma$. Now, since at that moment, the moon has to be at the horizon, the moon must also be at $\gamma$. Then, also, its declination is least.

[1] A simple inspection of a diagram will show that an element of a parallel will subtend the least angle at the pole when the parallel coincides with the equator.

We conclude, accordingly, that (assuming the moon to move along the ecliptic) its retardation is least (as seen by an observer in northern latitudes), whenever the moon is passing through $\gamma$ at the East point.

If now the sun is at Libra, at the same time, *i.e.*, at autumnal equinox, the moon is full. In other words, the full moon at the autumnal equinox undergoes least retardation or rises directly after sunset, for several nights in succession.

As this is helpful to harvest in Northern Europe, full moon at autumnal equinox is, in these parts, called "**harvest moon.**" Similarly full moon at the vernal equinox is Harvest moon for the Southern Hemisphere.

### Physical Features of the Moon.

26. The moon's surface presents features which are altogether different from what the earth would present to an observer, outside the earth. The surface is extremely variegated, being thickly interspersed with huge craters with but few long mountain-ranges such as are to be met with on the surface of the earth. Moreover, the mountains are comparatively high,—10000 to 20000 ft. being very common—having regard to the small size of the moon, in comparison with that of the earth.

The lunar craters, as a class, are nearly circular and are surrounded by a ring of mountains. In most cases, they resemble terrestrial volcanic structures, whence, it has been concluded that they have had a similar origin. Opinions however differ as to whether the lunar surface does not or does show any sign of volcanic or other activity. But, on the whole, recent consensus of opinion is in favour of the latter view. If this be so, these craters are very probably associated with such activities.

Objects on the moon's surface differ so much in appearance on account of the varying illumination

due to the sun, that it is difficult to be certain of these changes.

Besides mountains and craters, there are on the surface of the moon, many deep narrow crooked valleys called "rills" and "clefts," of the nature of fissures, in the lunar crust. Finally, we have probable evidence of radiation from certain of the craters, the so-called "rays," which appear like light-coloured streaks but the nature of which has not as yet been satisfactorily made out.

It will appear from even this brief account that our knowledge of the moon's surface of the portion, which alone is turned towards us is much fuller and much more accurate than in many portions of the earth, say in Asia and Africa, which have not been surveyed at all, so far.

### MEASUREMENT OF THE HEIGHTS OF LUNAR MOUNTAINS.

27. One of the simplest methods consists in measuring by means of a micrometer, the angular distance between the projection of a mountain top on the dark background of the non-illuminated portion of the disc from the "*terminator*" (*i.e.*, the line of demarcation between the light and dark portions) of the moon.

Let B (fig. 56) be the top of a mountain. It will appear as a star on the lunar disc, as the first rays of light from the sun (S) are caught by it, touching the moon at A.

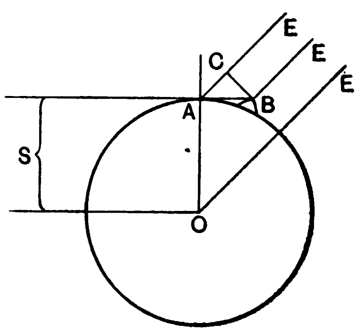


Fig. 56.

And if AE is the direction of the observer and the projection of B on AE is C, then the angular distance measured is the angle subtended at E by BC.

Thus, we have AB $\sin\theta = BC$.

where $\theta$=angle EAB=the elongation of the moon, and $AB^2=(2r+h)h=2rh$ nearly,[1]

if $r$=the radius of the moon and $h$ the height of the mountain. $\therefore BC=\sqrt{2rh}\sin\theta$, nearly.

Hence, since BC, as *an angular measure* is known, $h$, *as an angular* measure is determined, since $r$ and $\theta$ are known.

In order to convert these to actual distances, we have to multiply throughout, all the angular measures (expressed in circular measure) by the distance of the moon from the earth.

28. From careful observations, it has been concluded that the moon's atmosphere is either non-existent or is of extreme tenuity. For, looked at through the telescope, the edge of the moon is seen without any distortion or haze. There is, in fact, no evidence of any atmospheric phenomena. Further, when the moon comes between the observer and a more distant object, at an occultation of a star for instance, the phenomenon observed is instantaneous and not gradual, as would have been the case, if the moon had any atmosphere.[2]

29. It follows, also: that there cannot be any water on its surface, except in the form of ice, at sufficiently low temperature, to allow of its existence in that form, without evaporation into a practical vacuum. The surface of the moon, in fact, must be at an extremely low temperature, a conclusion which follows from the simple consideration that the lunar night extends over fourteen days, during which the temperature of the surface must fall very low indeed, while the long day of fourteen days cannot provide adequate compensation, as, on account of a practical absence of an atmosphere, the heat received from the sun must radiate away almost as quickly as received.

[1] In order to see this more clearly, draw the diameter through B. In fact, $OB=r+h$.

[2] Prof. Pickering is, however, of opinion that the moon has an atmosphere.

## EXERCISE.

1. Explain why the moon is regarded as a satellite of the earth.

2. Determine the position of the moon at which its phase will be $\frac{1}{8}$ (assuming the orbit to be circular).

3. Show, by means of a diagram, how the phase is modified, if we take account of the eccentricity of its orbit.

4. Is the phase independent of the position of the observer ? Show how it depends on his position by means of a diagram.

5. Assuming the motion of the moon to be uniform in its orbit, find its daily retardation. What is its amount (1) in a lunar day, (2) in a solar day ?

6. Show by means of a diagram, how the inclination of the horizon to the ecliptic changes throughout the year, at any place.

Hence explain the phenomenon of "Harvest moon."

7. How has it been concluded that the same face of the moon is always turned towards the earth ?

Is it absolutely correct ? What information is deducible from this ?

8. If the axis of the moon were perpendicular to the lunar orbit, find the region that will come within the earth's view, if the angular distances and the angular rotations of the moon during a certain interval are $90^\circ$ and $91^\circ$.

9. Find the height of a lunar mountain, given, Angular distance of a bright speck from the edge $2''$. The elongation of the moon $30^\circ$. The mean distance of the moon from the earth = 23800 miles.

10. Prove that at the end of every 19 years, the phases of the moon recur in the same order, as regards dates. (Metonic cycle.) [$29{\cdot}53059 \times 235 = 19$ years.]

11. If the sidereal period of the moon were 30 days, find what would be the length of one lunation.

12. The sidereal period of the moon is $27\frac{1}{3}$ days. The synodic period of regression of the lunar nodes is 346·62 days. Hence show that a period of 6585 days constitutes a cycle during which the relative positions of the sun, the moon and the earth as well as the lunar nodes recur in the same order.

13. Find the eccentricity of the lunar orbit, given that the greatest and least angular diameters of the moon are as 253 : 221·5.

14. Given that the mean distance of the moon is 239000 miles, find its diameter, if the mean angular diameters is $31'$.

15. T..e apparent diameter of the moon ranges between $33'\ 30''$ and $29'21''$. Assuming that the orbit is an ellipse, find its major axis and show how to trace t he orbit (round the earth).

## CHAPTER VII

### The Planets

1. While the circular paths of the sun and the moon among the stars, as traced on the celestial vault presented a comparative simplicity, which made a geocentric explanation of their motions by no means incredible, those of the planets were so complex that they baffled, for ages, the most ingenious attempts to decipher them. When the paths of planets, such as Mars is traced on the celestial sphere by means of their observed R.A. and declination, as in the case of the sun and the moon, it is found that, although they never lie very far away from the ecliptic, they do deviate North and South (as much as 8° in some cases) and shew, moreover, loops and kinks as in Figure 57.

2. Comparing these motions with that of the sun, we find, that a planet appears sometimes to move in the same direction as the sun; the motion is then said to be **direct**. At other times, it appears to move in the opposite direction, when the motion is called **retrograde**, while there are occasions, when they seem to stand still (relatively to the sun) or to be **stationary** [Art. 14.]

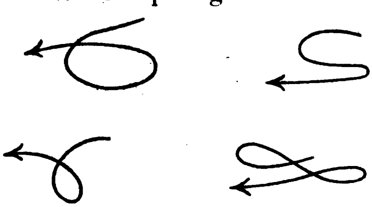


Fig. 57.

3. The most important problem of ancient astronomy was naturally to explain these complicated motions, which appeared well nigh to defy analysis.[1]

[1] The first step in analysis was made in remote ages. This was to classify the complicated motions they appeared to possess. The

4. Among the first attempts at analysis were those made on the dictum of Plato (425 B.C.) that the circular motion was the perfect motion, and on this basis, a theory of celestial appearances was devised by Eudoxus (408-355 B.C.). The problem he attempted to solve was, so to combine uniform circular movements, as to produce the resultant effects, actually observed. The sun, the moon and the [five] planets were, with this end in view, accommodated, each with a set of variously revolving spheres, to the total number of twenty-seven. This was modified by Appollonius of Perga by means of the hypothesis of epicycles (Intro. 18), which held the field for 1800 years, as the one generally accepted theory of planetary motions.[1]

5. In order to realize in what manner, this highly artificial theory of epicycles failed, it is necessary to understand the nature of the problem which it attempted to solve :

following classification of these motions in the *Suryya Siddhanta* is remarkable, as being based on an exhaustive survey of the entire motion.

1. *Vakra*—decreasing retrograde motion.
2. *Ativakra*—increasing retrograde motion.
3. *Kutila*—stationary position.
4. *Manda*—increasing direct motion, less than the mean motion.
5. *Mandâtara*—decreasing direct motion, less than the mean motion.
6. *Sama*—mean motion.
7. *Sighra*—decreasing direct motion, greater than the mean motion.
8. *Sighratara*—increasing direct motion greater than the mean motion.

[1] Another worker in the same field was Hipparcus, who fixed the length of the tropical and siddereal years, of the various months and of the synodic periods of the various planets, determined the obliquity of the ecliptic and of the moon's path, the position of the sun's apogee and the eccentricity of his orbit and the moon's horizontal parallax. He made use of eccentrics which accounted for changes in the orbital

In the infancy of the astronomical science, the observer, naturally regarding the earth to be fixed, looked on the Sun, the Moon and the planets as occupying domes, arranged, one above the other, the "Heaven of the Moon," the "Heaven of Mercury," etc., which the moon, the mercury, etc., were respectively supposed to inhabit and in which they completed their sojourn in paths, which were assumed to be circular. When, however, some progress had been made in accurate astronomical observations, it became evident that such a simple theory could not explain the motions of the planets, although they explained those of the sun and the moon, fairly well. Astronomers, accordingly, sought to explain these motions, by imagining the sun and the moon to be in motion round the fixed earth and the planets round the sun. This was evidently the standpoint of *Suryya Siddhanta,* as the motions of the planets given there are those round the sun. This was also the system of Tycho Brahe. According to him: "The earth is the centre of the universe and the centre of the orbits of the moon and the sun, as well as that of the sphere of the fixed stars. The sun is the centre of the five planets, of which Mercury and Venus move in orbits whose radii are smaller than the solar orbit, while the orbits of Mars, Jupiter, and Saturn encircle the earth." Now if the orbits of the planets were actually circular and in the plane of the earth's orbit, such a hypothesis would have

velocity of the sun and the moon by a displacement of the earth from the centre of the circles they were assumed to describe.

It is noteworthy that in the *Suryya Siddhanta*, also, rules are given for determining the position of the planets at any time, which fairly agree with the then observed places. These give successive approximations-*manda phala* (1st equation), *sighraphala* (2nd equation of 1st, 2nd and higher orders). These are also based on epicycles, though it is not clear how far the systems of epicycles used here agree in character, with those used by the Greeks.

fairly adequately represented these motions. But the orbits are neither circular nor co-planar. Hence, the necessity for the introduction of epicycles of various degrees of complexity and the various corrections, the most complete exposition of which seems to be contained in the Almagest of Ptolemy. At the same time, it is reasonable to suppose that astronomers at different times tried to free themselves from the highly artificial theory of epicycles, which was intimately associated with a geocentric scheme. Accordingly, the other, the heliocentric view was revived from time to time. It is, indeed, conceivable that the germ of this idea could be traced to the early thinkers. It is contended that the Hindus knew, at any rate, that the geocentric theory could not explain planetary motions. It is certain, however, that Pythagoras (569-470 B.C.) (who is said to have come to India to study Mathematics) or one of his followers propounded a system, somewhat similar to the one accepted now.[1] The idea of the earth's motion round a central body with the further modification that the sun is this central body was revived by Copernicus,[2] in the Sixteenth century but he also attempted to explain all motions, as made up of circular motions. The problem was thus practically unsolved, at the time Kepler took it up.

6. Kepler was an assistant of Tycho Brahe and came into possession of the latter's splendid results after his death. It was after vainly attempting to fit in these results,

[1] The centre of the universe is occupied, according to the Pythagoreans, by the central fire as the hearth of the universe round which the earth and all the heavenly bodies move in circular orbits.

[2] This is what Copernicus says: We are not ashamed to maintain that all that is beneath the moon with the centre of the earth describe among the other planets a great orbit round the sun, which is the centre of the world; and that what appears to be a motion of the sun is, in truth, a motion of the earth.

the accuracy of which was undoubted, with a hypothesis of epicycles of increasing degree of complexity, that he gave up the postulate of the stationary earth and adopted the hypothesis of a moving earth—moving about the sun. But he not only postulated a helio-centric system but, from a detailed analysis of the results of Tycho's observations, specially on the motion of Mars, showed that the orbits of the planets are ellipses, variously inclined to each other and to the ecliptic, with the sun at one of the focii. He, in fact, deduced from these observations, his three celebrated laws, embodying a remarkably complete and coherent scheme:

7. These laws are—

(1) The orbits of the planets are ellipses (with the sun at one of the focii).

(2) The radius vector joining the sun to a planet sweeps out equal areas in equal times.

(3) The square of the time taken by a planet to complete its orbit is proportional to the cube of its major axis.[1]

8. At the present day, we have direct confirmation of Kepler's hypothesis—supplied by the phenomena of aberration, discovered by Bradley. Bradley observed that when star places were accurately noted, they *all* appeared to describe small ellipses, parallel to the ecliptic, and to complete a cycle in a year. It was, therefore, *a priori* evident that the observed motion was apparent motion and could only be due to the motion of the observer, carried by

[1] It is not without interest to note that the second law was deduced by Kepler, as the result of two mistakes. He took it for granted that the heliocentric motion of a planet is due to force, directed towards the sun but varying inversely as the distance of the planet from the sun. Hence, according to Kepler, the velocity is inversely proportional to the radius vector and, consequently, the time required to describe the corresponding arc is also proportional to the radius vector—in a circular orbit and, therefore, in all orbits. Accordingly, the total period will be

the earth in its motion round the sun. In fact, if the earth were at rest, a star would be seen by means of light issuing from it and coming to the observer with a known velocity—in the direction in which the light actually comes. If, however, the observer is in motion, the direction in which the star will be seen is the apparent direction in which light *appears* to come, just in the same way as to a man walking forwards, a rain drop falling vertically appears to come towards him in a slanting direction. Actual calculation shows that this exactly accounts for the displacements observed and, thus, aberration supplies practically an ocular demonstration of the earth's motion.

9. Remembering now that the earth rotates about an axis whose direction is fixed in space, the complete motion of the earth can be described by means of the following model :—

Imagine the plane of the ecliptic to be an inclined plane, suitably inclined to the horizontal plane, assumed to coincide with the equator. Then, the line of equinoxes will coincide

proportional to the sum of all the radii. And *this*, he took to be equal to the total area described.

It should be admitted, however, that, although Kepler obtained the law in this accidental manner, he finally adopted it, only because it agreed with the results of observation.

But if the discovery of this law was accidental, the same is not true of the law of elliptic orbits. Continued observations of Mars by Tycho Brahe had accumulated data, from which Kepler set himself the task of deducing the nature of its path. Since the circular orbit, even an eccentric one, had to be given up, as the result of observations abundantly showed, he concluded it to be an oval curve of some kind, agreeably to the law of equable description of area ; but as the simplest oval is an ellipse, he attempted to work on the hypothesis of such an orbit and found his hypothesis justified on the ample data available. Thus, the problem of planetary orbits was solved in its kinematical aspect and Kepler showed that no other hypothesis could be made to accurately agree with observations.

with the horizontal line passing through the sun's centre and the line of solstices, with the line of greatest slope. The earth's centre will then describe an ellipse, on this (inclined) plane, with the sun's centre at one of the foci, while its axis of rotation will always remain vertical.

10. If we admit that it is the earth that moves round the sun, then these complicated motions of the planets are found to be due only to motion round the sun, as observed from the moving earth. The system of planets, *viz.*, Mercury, Venus, etc., including the earth, forms, on this view, a **heliocentric** system. It has been, moreover, found that the distances of the planets from the sun increase, as we pass for Mercury to Neptune. (See Table). Accordingly, Mercury and Venus are called **inferior** planets (or nearer to the sun than the earth) and Mars and the rest (including the *asteroids*), **superior** planets.

11. Admitting this, the heliocentric, view of Kepler, *viz.*, that the planets, including the earth are in motion about the sun and that the sun is fixed in space, let us follow out the consequences of such a view. In order to simplify the reasoning, let us assume that the orbits of the planets about the sun are circular and coplanar.

Then, if P be an inferior planet (fig. 58) its angular velocity about the sun is greater than that of the earth (Kepler's 1st law). If we, therefore, superpose at any moment, an angular velocity equal and opposite to that of the earth, on the whole system, the resultant motion will be that relative to the earth, at the moment—the sun moving round the earth with an angular velocity, equal and opposite to that of the earth and the planet round the sun, with *reduced* angular velocity.

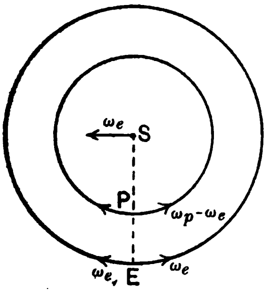


Fig. 58.

12. This is virtually the Tychonic system and this mode of representation would have the observed motions, if the supposition made above (that the orbits are circular and coplanar) represented the actual facts.

13. Moreover, when P is between the earth and the sun, it will appear to be moving in the same direction as the sun, while when the sun is between the planet and the earth, it will appear to be moving in the opposite direction. The former is, as we have seen, called **retrograde** motion, the second, **direct** motion and it is clear that there will be two positions, at which the motion changes from direct to retrograde and *vice versá*. At these positions, the motion is neither retrograde nor direct; that is, the planet will not be at all changing its angular position relative to the earth. In other words, it will appear to be **stationary**.

14. These characteristics of the motion of an inferior planet relative to the earth are well-known (art. 2) and are thus sufficiently explained on the heliocentric view.

15. Now, since these motions are relative, to an observer in P, the motions of the earth will be similar to those of P relative to the earth—*viz.*, direct, retrograde and stationary, corresponding to retrograde, direct and stationary motion of P relative to E.

The motion of a superior planet, therefore, will, also, have these same characteristics, relative to the earth.

16. *Def.* **Elongation** of a planet is the angle subtended at the earth by the line joining the planet and the sun.

When the elongation is zero, the planet is said to be in **conjunction—superior,** when the sun is between the planet and the earth and **inferior,** when the planet is between the sun and the earth.

Obs. It is only an inferior planet that can be in inferior conjunction, while both inferior and superior planets can be in superior conjunction.

When the elongation is 180°, the planet is said to be in **opposition**.

E = The Earth.

S = The Sun.

P = Inferior planet.

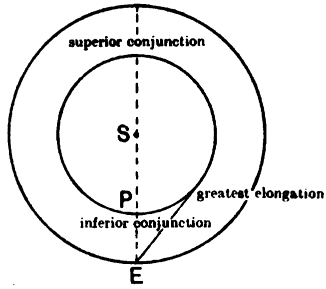
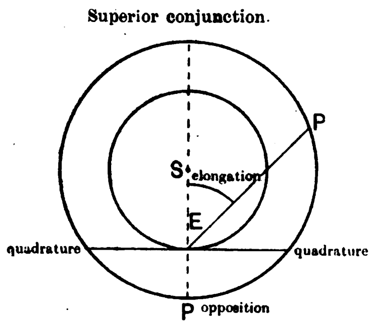



Fig. 59.

When the elongation is 90°, the planet is in **quadrature**. [Fig. 60.]




S = The Sun.

E = The Earth.

P = Superior planet.

Fig. 60.

It is only a superior planet that can be in opposition or in quadrature, while the elongation of an inferior planet must be between 0° and a certain maximum value less than 90°. [Fig. 59.]

The maximum elongation of mercury lies between 18° and 28°.

That of Venus is 47°.

17. As the elongation of a planet measures the angular distance of the planet from the sun, *as seen from the earth*, an inferior planet can never be, *relatively*, very far from the sun, while a superior planet can be at all angular distances from it.

It follows, therefore, that an inferior planet will rise or set shortly before or shortly after sun-rise or sun-set, while a superior planet may rise or set at all times.

An inferior planet is therefore either a "morning star" or an "evening star."

Let the arrow head represent the sun's apparent annual motion as seen by an observer at E. [Fig. 61.] Then $V_2$ is the relative position of Venus at its Western elongation and $V_3$, at the Eastern. From $V_4$ to $V_3$ and on to $V_1$, therefore, Venus is a "morning star" while, from $V_1$, up to $V_2$, $V_4$, she is an "evening star."

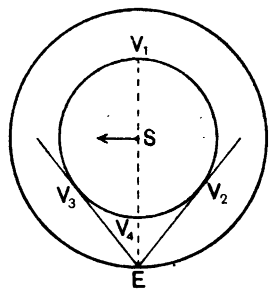


Fig. 61.

18. That the planets are opaque bodies is best proved from their phases.

*Def.* **Phase.**—The phase of a planet is measured by the portion of its illuminated surface, which is turned towards the earth.

19. As the phase, as well as the elongation, depends on the relative positions of the sun, the Earth and the planet, it will be sufficient if we consider the changes of phase, as the elongation changes; in other words, we may imagine the planet alone to move.

20. Taking, first, the case of an inferior planet, when the planet is in inferior conjunction, the whole of the illuminated portion is turned away from the earth. It is then said to be **New** (Ch. VI, 17).

The phase increases as the elongation increases.

At the maximum elongation, the phase is half.

But while the elongation decreases after that, the phase goes on increasing, till at superior conjunction, the planet is **Full**. The reverse is the case from superior to inferior conjunction.

In fact, if P is an inferior planet (Fig. 62), then the tangent at P to its orbit (assumed, circular) separates the illuminated from the dark portion of the planet.

Now, if E be the earth, and PE is joined, then the section of the planet perpendicular to PE separates the half presented to the observer from the half, turned away from him.

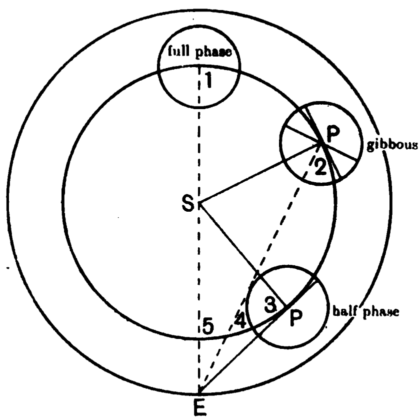



Fig. 62.

Accordingiy, since the phase is measured by the portion of the illuminated surface turned towards the observer it varies as $\frac{1}{2}(1+\cos \text{SPE})$. [Ch. VI, 17.]

It is easily seen that at an inferior conjunction, $\angle$ SPE $=180°$ and therefore, the phase $=0$.

At max. elongation, $\angle$ SPE $=90°$, or the phase $=\frac{1}{2}$ and at superior conjunction, $\angle$ SPE is equal to $0°$, or, the phase $=1$.

21. As the increase of phase is associated, in the case of an inferior planet with increased distance, such a planet (Venus for instance, since Mercury is hardly visible) does not present to the naked eye, any appreciable variation in brightness. In order to detect its phases, therefore, it is necessary to use a telescope. The figure (63) gives the telescopic view of Venus.

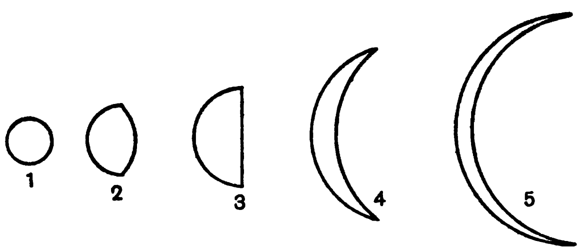



Fig. 63.

22. The variation in phase in the case of a superior planet presents a special feature which is worthy of note. In considering this, we recall in the first place that the phase varies as $\frac{1}{2}$ (1 + cos SPE).

But $\dfrac{\sin \text{SPE}}{\sin \text{SEP}} = \dfrac{\text{SE}}{\text{SP}}$ (fig. 64), if P is a superior planet and E, the earth. $\therefore \sin \text{SPE} = \dfrac{\text{SE}}{\text{SP}} \sin \text{SEP}$.

23. But the greatest value of sin SEP is 1, when SEP $=90°$. Or sin SPE is greatest at quadrature and cos SPE is least, at the same time. That is, the phase is least at quadrature and even then, the phase is more than half. It is then said to be most *gibbous*.

17

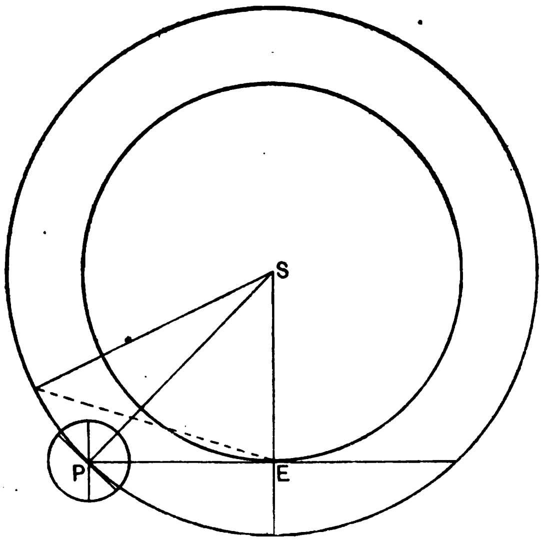



Fig. 64.

24. We have seen that the motions of the planets obey three laws, discovered by Kepler, by observation alone.

Following the order of their discovery the second law may be thus stated : If a planet goes from P to T, in one day and from T to U, the next day, then S being the Sun, the area SPT is equal to the area STU. [Fig. 64 A.]

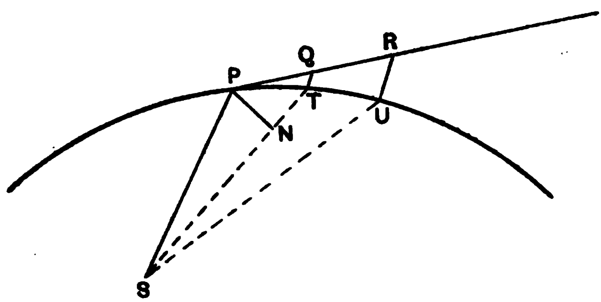



Fig 64 A.

Now, if the angle PST is small, the area SPT is proportional to the product, $SP^2$. $\theta$ where $\theta$ is the circular measure of the angle PST, for SP, or the distance of

the Earth from the Sun, to take the case of the *earth* first, varies inversely as the angular radius of the sun. and the angle PST is really (art. 26) the change in the celestial longitude of the Sun. Hence, by observing the angular diameter of the sun and its change in longitude from day to day, the law can be verified in the *case of the Earth*. In the case of any other planet, the reduction of observations is much more complicated, but the general principle is similar. [Art. 27.]

25. The first law states that these orbits are ellipses. The actual procedure in the case of the earth may be roughly sketched, as follows.

We have seen that the *apparent* orbit of the sun is an ellipse, with the earth at one of the foeii. From this, the *real* orbit of the earth can, at once, be deduced, since the sun is really at rest and the earth is in motion.

It will be remembered (ch. V, 17) that the sun's apparent orbit was obtained as a relation between ES and the $\angle$ SE$\gamma$ (fig. 39). Thus, if E is regarded as fixed and S, in motion, $\gamma$ES is the longitude of the sun and ES is the corresponding radius vector, S moving in the direction of the arrow towards S′. (Fig. 65.) [1]

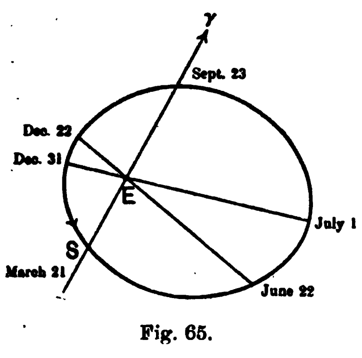



Fig. 65.

Apparent orbit of the sun.

Now, in the derived orbit of the earth (fig. 66), S is the fixed body and S$\gamma$, parallel to E$\gamma$ is a fixed direction in space. Then, taking SE, equal and parallel to ES and *similarly directed* in both diagrams (figs. 65 and 66) and SE′ = ES′ and parallel,[1] we observe that the motion of E is from

[1] E′, S′, consecutive points to E (fig. 66) and S (fig. 65) are not shown in these diagrams.

E to E′, *i.e.*, opposite to that of S and that the orbits are equal.

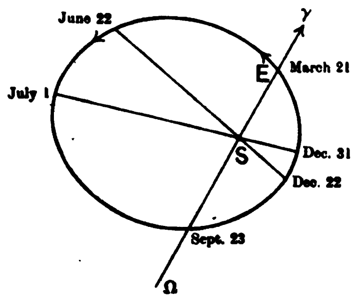



Fig. 66.
The real orbit of the earth.

*Note.*—$\overrightarrow{SE\gamma}$ is a fixed direction in space and, evidently, the change in the longitude of the sun is equal to the change in the angular co-ordinate of E. (Fig. 66).

In other words, since the radius vector and angular co-ordinate are the same, whether the sun or the earth is the fixed body, the *apparent* path of the sun will be exactly of the same nature as the path of the earth, with reference to the sun, the direction of motion in one case, being opposite to that in the other. The path is, thus, in either case, an ellipse with the fixed body at the focus of the ellipse.[1] This proves Kepler's 1st law of motion, in the case of the earth.

[1] The problem of the earth's motion, as it presents itself in its *entirety* is thus one of great complexity. On account, however, of the smallness of the eccentricity of the elliptic orbit, the solution can be effected by successive approximations. The actual angular position differs from the mean position by a small quantity, which is called the *equation of the centre* and depends on the eccentricity.

26. The Earth's path being thus determined, Kepler's second law can also be verified with reference to this orbit Referring to fig. 67 we have to prove that if the areas ASB, SCD, SEF, are described in equal times, they are equal. We have already seen (art. 24) how this can be done. (Observe that the ∠CSD, is the change in the longitude of the sun in the time taken by the earth to go from C to D. [Art. 25, note.]

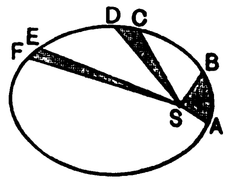


Fig. 67.
Equable Description of Areas.

*Note.*—It follows from this, that the seasons (Ch. V, 36, 37) are of unequal lengths, being proportional to the areas marked off by the line of equinoxes and solstices in figs. 65 and 66.

Further, following the data specified in Ch. V, 27, we may describe the *motion of the earth, as follows*:—

The earth is in perihelion on 31st December and in aphelion on July 1st.

It is at ♈ on 21st March and at ♎ on September 23rd. [Fig. 66.]

27. The path of any other planet, with reference to S is more difficult to determine.

In order to illustrate the method, we shall assume the orbits to be circular and in the plane of the ecliptic.

Let P (fig. 68) be the position of an Inferior planet and E, the Earth at inferior conjunction and let P′, E′, their positions at any other time. Then, we know the ∠PSE′ and SE′, being the corresponding vectorial angle and the radius-vector of E. Also, the elongation of the planet, *viz.*, SE′P′ can be determined, as well as the angle E′SP′, or the angle [1] gained by the planet on the earth

[1] If $t$ is the time from E to E′, the circular measure of this angle is $\frac{2\pi}{S}t$, where S is the synodic period.

round the sun. Thus, the triangle SE′P′ can be solved and the length SP′ or the radius of the orbit can be determined.[1] Also when SP′ and the diurnal change in the ∠ ESP′ is known, the second law can be verified, as in the simpler case of the earth. [Art. 24.]

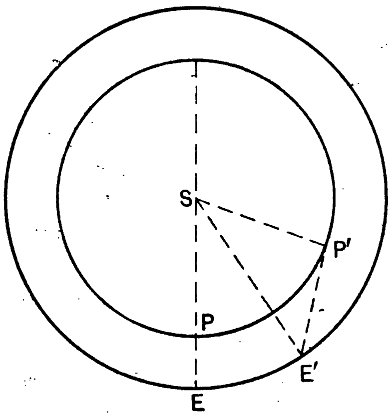


Fig. 68.

28. The third law states that the squares of the periodic times vary, as the cubes of the major axes or the greatest diameters of the elliptic orbits, described by the planets.

Since the orbits are known, all that has to be done is to calculate the time taken by a planet to go round the sun, that is, its sidereal period. The formula (Ch. VI, 13) enables us to do this. We may state the argument in a different form. For this, we notice, that if the Earth goes round the sun in E days, it goes $\frac{1}{E}$ of the total circuit (round the sun) in one day; similarly, the planet goes $\frac{1}{P}$ of the circuit (round the sun) in one day, if its periodic time is P; thus, it goes

[1] If, instead of calculating the angle E′SP′, on the assumption that the planet's orbit, is circular, we measure the angular diameter of the planet (as seen from the earth), we know the distance E′P′, on an assumed scale. This would enable us to solve the triangle SE′P′ and determine SP′, as well as the angle ESP′. Thus, the path can be determined, and proved to be an ellipse. This, however, still involves the assumption that the orbit of the planet is in the plane of the ecliptic.

ahead of the Earth by $\left(\frac{1}{P}-\frac{1}{E}\right)$ of the circuit in one day. When it has gone ahead by a whole circuit, it will be in the same relative position to the Sun and the Earth as at the beginning. The interval (the *synodic period*) is, therefore, $\frac{EP}{E-P}$. This interval can be observed—for it is the interval between, say, two successive conjunctions, and hence, P can be determined, since E, the number of days in the year, is known. In the case of a superior planet, the synodic period is $\frac{EP}{P-E}$.

29. These laws seem, at first sight, to be too complicated to be of much use. It is remarkable, however, that they are all consequences of a single law—the law of universal gravitation.

30. *And we reach this grand generalisation—that all these complicated motions of the planets are due to an attraction (directed to the sun), varying according to the law of inverse square of the distance or the acceleration, under which a planet P moves in its orbit is* $\mu$ $SP^2$, *where S is the centre of the sun and,* $\mu$, *a constant (nearly) for all the planets.*

31. Let P be the position (fig. 64 A) of a planet at any time, moving in the direction PQ. If there had been no action of the sun, the planet would have continued to move along PQ, and Q, R would have been its positions at $t$, $2t$ seconds after ($t$ being small).

On account of the attraction of the sun, however, directed along PS, it occupies positions marked T, U instead. In other words, the planet is displaced through QT, RU, in the times $t$ and $2t$ seconds. Since these

displacements are due to a force acting along PS, very nearly, QT, RU are parallel to PS, and the figures PST, PSU are, ultimately, triangles. We have, accordingly, by Euclid, the triangle PQS, (QS being joined)

=the triangle PST.
=QSR.
=TSU.

That is, the areas PST, TSU, are equal or the area described by a planet in any time is proportional to the time. Thus, the second law of Kepler is seen to flow from the supposition that the motion of a planet is due to an attractive *force directed to the sun.*

32. Again, if V is the velocity of the planet at P,

$$PQ = Vt, \qquad (1)$$

and if $p$ is the perpendicular from S on PQ,

$$\text{we have } PQ.p = Vt.p. \qquad (2)$$

which as we have just proved, is proportional to $t$.

$$i.e., \ Vp = \text{constant}.$$

Calling this const. $h$, we have $Vp = h$ (3)

$$i.e., \ V^2 = \frac{h^2}{p^2}.$$

But the kinetic energy, acquired by a particle is equal to the work done by the force producing the motion.

Now, as the planet is displaced from P to T, T being a neighbouring point, the displacement may be conceived to be made up of PN (perp. to ST) and TN. The first, being perpendicular to the force (along ST) produces no work. The work is, therefore, measured by the product, F. TN, where F is the force at T.

If, then, U is the velocity at T, V being the velocity at a *specified point* P, we have, remembering that $F = \mu\, SP^2$,

$$\tfrac{1}{2}(U^2-V^2)=-\frac{\mu}{ST^2}\,TN=\mu\frac{SP-ST}{ST.\,SP},\text{ very nearly,}$$

$$=\frac{\mu}{ST}-\frac{\mu}{SP},\text{ very nearly;}$$

$$\text{or, } U^2=\frac{2\mu}{ST}+V^2-\frac{2\mu}{SP}.$$

But $U^2=\frac{h^2}{p^2}$, where $p$ is the perpendicular from S, on the direction of motion at T.

$$\therefore\ \frac{h^2}{p^2}=\frac{2\mu}{D}+V^2-\frac{2\mu}{SP}=\frac{2\mu}{D}\pm\text{ constant,}$$

where D = distance of the planet from the sun, at any point and $p$ = perpendicular from the sun on the direction of motion, at that point.

But in a conic section,

$\frac{2}{D}$ is greater than, equal to, or less than $\frac{l}{p^2}$, where $l$ is the semi-latus rectum, according as it is an ellipse, a parabola or a hyperbola.

Hence, the path of a planet is a conic section where $h^2=\mu l$ and it is an ellipse, provided it has the required velocity at any particular point; that is, provided $V^2$ is *less* than $\frac{2\mu}{SP}$, where V is the velocity at any point P. As the body has always the same velocity, whenever it comes to the same position, it must have started, at some unknown point of time, with a certain definite velocity which was suitable for the description of the elliptic path.

33. From (2) and (3) we have

$$PQ.p=2\text{ area }PST=ht.$$

*i.e.*, twice the area described in time $t=ht$.

$\therefore$ the whole time K of describing the ellipse is given by $h\mathrm{K}=2$ area of the ellipse.

Now, $l=\frac{b^2}{a}$ and the whole area of an ellipse $=\pi ab$, where $a$, $b$ are the semi-axes of the ellipse.

$\therefore$ $\mathrm{K}^2=4\frac{\pi^2a^2b^2}{h^2}=4\pi^2\frac{a^3}{\mu}$ which is Kepler's third law.

34. Since the orbits are, in general, ellipses of small eccentricity, we may regard them as circles, for rough calculations. Assuming this to be the case, we may prove the 3rd law more simply, as follows. [The sun is, then, to be regarded as a *fixed* body, placed at the centre of the circle.]

Force, directed to the centre $\propto\frac{\mathrm{P.S}}{r^2}$

This must be $=\frac{\mathrm{P}v^2}{r}$,

where P = mass of the planet
S = „ sun
$r$ = radius of the circle
$v$ = velocity in the orbit.

Since, moreover, there is no tangential force, $v$ is constant $=\frac{2\pi}{\mathrm{K}}$, K being the periodic time.

Hence, $\mathrm{K}\propto r^3$.

## Habitability of the Planets.

35. The question of the habitability of the planets is one, naturally, of great interest. No information, however, on this point is available at present. The markings on Mars called canals, for instance, cannot tell us anything as to whether Mars is inhabited, for it is too far off for us to be able to understaud the true nature of these markings.

The reason is obvious. The distance of Mars from the earth is about 200 times that of the moon. With a telescope, therefore, having a magnifying power of 200 (nearly the best magnifying power, we possess) we can see Martian objects, as clearly as we can see lunar objects with the naked eye. That, at once, explains that our attempt to find out the true nature of objects on Mars must fail, till we have telescopes of much greater magnifying power, than we have any hope of making at present. But there is a further limitation. With a telescope of high magnifying power, there will be a corresponding diminution of light and definition, so that it seems to be unlikely, that it will ever be possible for us, constituted as we are, to get any direct information regarding Mars, except of the vaguest kind.

36. In the following table will be found all the more important points referring to the solar system. Any detailed discussions of these are beyond our scope. There are certain facts, however, which will easily appear to be especially worthy of note.

(*a*) *Inclination of the orbits.* The orbits of the planets are all inclined to the ecliptic, at comparatively small angles, except Mercury, whose inclination is $7^{\circ\prime}\ 0'\ 10''$ and some of the asteroids (Pallas, at 34°).

(*b*) *Eccentricity of the orbits.* This quantity is small in all cases, except in that of Mercury, whose eccentricity is ·205, that of the earth being ·016.

(*c*) *Rotations.* It is reasonable to suppose that all the planets rotate about axes, more or less inclined to their orbits. But nothing is known for certain, about Mercury or Venus—which are lost in the solar rays nor about Uranus and Neptune which are too far off. Of the rest, the period of rotation of Mars is very nearly equal to that of the earth, while those of Jupiter and Saturn are

## Table of Names, Distances, Periods, etc.

| Name. | Symbol. | Distance. | Bode. | Diff. | Sid. Period. | Syn. Period. | Eccentricity. | Incl. to Ecliptic. | Time of Axial Rotation. | Number of Satellites. |
|---|---|---|---|---|---|---|---|---|---|---|
| Mercury ... | | 0·387 | 0·4 | −0·013 | $88^d$ | $116^d$ | ·2056 | 7°0′8″ | | |
| Venus ... | | 0·723 | 0·7 | +0·023 | $224·7^d$ | $584^d$ | ·0068 | 3°23′35″ | | |
| Earth ... | | 1·000 | 1·0 | 0·000 | $365\frac{1}{4}^d$ or $1^y$ | ... | ·0168 | 0°0′0″ | $23^h·56^m·4·09^s$ | 1 |
| Mars ... | | 1·523 | 1·6 | −0·077 | $687^d$ | $780^d$ | ·0933 | 1°51′2″ | $24^h·37^m22·67^s$ | 2 |
| Mean Asteroid | | 2·650 | 2·8 | −0·150 | $3^y·1$ to $8^y·8$ | various | | | | |
| Jupiter ... | | 5·202 | 5·2 | +0·002 | $11^y·9$ | $399^d$ | ·0482 | 1°18′42″ | $9^h55^m$ | 4 |
| Saturn ... | | 9·539 | 10·0 | −0·461 | $29^y·5$ | $378^d$ | ·0560 | 2°29′40″ | $10^h14^m24^s$ | 8 |
| Uranus ... | | 19·190 | 19·6 | −0·410 | $84^y·0$ | $370^d$ | ·0470 | 0°46′22″ | | 4 |
| Neptune ... | | 30·070 | 38·8 | −8·730 | $164^y·8$ | $367\frac{1}{2}^d$ | ·0083 | 1°46′45″ | | 1 |

very nearly equal to each other, being less than half that of the earth.

Moreover, the inclination of the axes of rotation on which the seasons depend are much like that of the earth, in the case of Mars and Saturn, while that of Jupiter is very small (3°4′0″), so that Jupiter has practically no seasonal changes.

(*d*) *Satellites.* Even if Mercury and Venus had any satellites, they would be lost to view, on account of the sun's rays. At a total eclipse, however, under favourable circumstances, they might have been visible. No such object has so far been observed, though it cannot be asserted with certainty that no such object exists.

The discovery of a new satellite (as well as of a new planet), with improved means of observation, is in fact always, a possibility. In the case of Mars, for instance, two small satellites were discovered by Mr. Hall of Washington in 1877, during opposition, when the planet was very close to the earth.

The other planets have all one or more satellites, the earth having one (the Moon), Jupiter, four, Saturn, eight, Uranus, four, and Neptune, one. The motion of all of them obeys Kepler's laws, and is direct, except that of the satellites of Uranus and Neptune, which is probably retrograde. Their times of revolution vary, being, however, in every case, greater than the period of rotation of the primary (in those cases, in which the latter has been observed), except in the simple case of Phobos whose sidereal period is $7^h\ 39^m\ 15^s$.

(*e*) *Physical features.* On account of the nearness of *Mercury* and *Venus* to the sun, the details of their features have not been made out with any certainty. The evidence of their having an atmosphere is available but is not conclusive. The most striking facts about them are their

transits across the sun's disc, that of Mars being necessarily less so, than that of Venus.

The appearance of *Mars* presents features which have a remarkable family likeness to those of the earth, the most conspicuous of which is the polar "ice-caps," which are brilliant white patches and which, from their seasonal appearance and disappearance have been taken to be masses of ice. But the evidence, as to the real nature of the markings on Mars is still lacking. It seems, however, reasonable to suppose that Mars have an atmosphere. Moreover, on account of the fact that the obliquity of the planet's orbit to its equator is 24°, the conclusion is justified that its seasonal changes are similar to those of the earth, but owing to great eccentricity of its orbit, the lengths of the various seasons vary among themselves, considerably more than on the earth.

Considerably less is known about *Jupiter*, the largest of the planets, than in the case of Mars. The most striking telescopic feature is presented by the so-called belts, which appear as dark streaks on a bright background and are probably belts of the planet's atmosphere, which is very thick, corresponding to the trade-winds, in the earth's atmosphere.

*Saturn.* The most interesting planet from the telescopic point of view is, undoubtedly, Saturn on account of the so-called rings. These, three in number, consist of an infinite number of meteors, coursing round the planet, as its satellites.

*Uranus*: The above complete the list of the planets known to the ancients. Uranus, Neptune and the asteroids are modern discoveries. On March 13th, 1781, the elder Herschell discovered a new celestial object, which had a motion of its own and which he took to be a comet. It was soon found however, that it moved in a (nearly)

circular orbit and, as such, was to be regarded as a planet. This was the planet Uranus.

*Neptune.* When the orbit of the planet Uranus was worked out from observed data, it was found to deviate from its calculated path, by an amount, which could not be explained, otherwise than by supposing it to be due to the perturbations of an unknown planet. This led to the discovery of the planet Neptune. [Intro., 24.]

37. *Bode's Law.* When the distances of the first six planets from the sun, namely, those of Mercury, Venus, the Earth, Mars, Jupiter and Saturn, are noted, it will be seen that these are roughly in the ratio of 4, 7, 10, 16, 52, 100. Now, if we write down the numbers 0, 3, 6, 12, 24, 48, 96, and add 4 to each, we get the above numbers, except that there will be one isolated number 28, with no planet, corresponding to it. When, therefore, on the discovery of Uranus, its distance was found to obey that law also, its distance being found to be proportional to $96 \times 2 + 4$, an organized effort was made to search for a planet which would fill the supposed gap. As the result of this search, a planet was discovered and named *Ceres,* which fairly well answered to the criterion. Since then, however, a very large number of very small planets have been discovered, situated between the orbits of Mars and Jupiter, which, on account of their smallness—the largest of them being only about 228 miles long—are called **asteroids** and which seem to be fragments of a parent planet, as the result of an explosion. As some confirmation of such a hypothesis, it may be stated that the orbits of some of them resemble those of comets in their eccentricity and obliquity. Planets lying between the sun and the asteroids are called **interior** planets. Those lying beyond the asteroids are called **exterior** planets.

38. Comets or hairy stars are easily recognisable, on account of their appearance, in which it is easy to distinguish a brighter portion, the *coma* or *the head* with, in some cases, a still brighter nucleus, and, generally, the *tail*, flowing from the head and containing a less bright portion, which is always turned away from the sun. There is reason to believe that they are white-hot masses of gas highly attenuated, increasing in brilliance, as they approach the sun, on account of increased velocity. In some cases, the comet has the appearance of a gas-jet, issuing from the nucleus, indicating a violent internal commotion, due, it may be, to the action of the sun and resulting in a loss of material.

39. The motion of a comet is also another of its distinctive features. The path (fig. 69) is, in general, a conic section of large eccentricity, which, in most cases, is practically unity. Thus, the path is an elongated ellipse, which in many cases becomes parabolic. In some few cases, the eccentricity has been found to be even greater than one, so that the paths are hyperbolas.

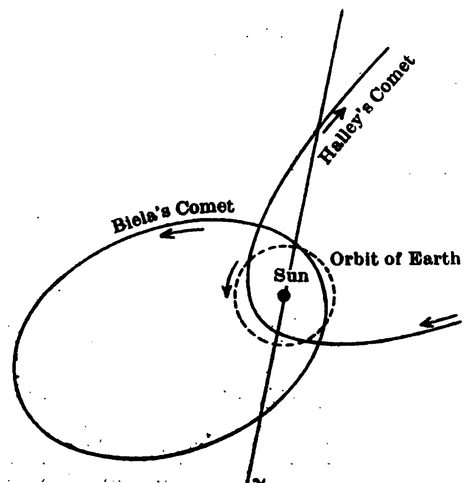



Fig. 69.

40. In those cases, in which the eccentricity is not too near unity, the comet is said to be periodic, as it returns to the region of the solar system, which is within our purview.[1] The most famous of them is Halley's comet which has a period of about 75 years. The orbits of these comets are all inclined to the ecliptic, at considerable angles. The fact that the paths of the comets are conic sections indicate that they belong to the solar system. It may, however, be that the majority of them are moving in space and *come* to describe the paths, that we actually observe, *when* they enter the solar system—on account of the forces that *then* become operative.

## EXERCISE.

1. The maximum elongation of a planet is 30°. Is it an inferior or a superior planet? Find the radius of its orbit, assumed to be circular and in the plane of the ecliptic.

2. A planet is found,

   (*a*) to have an elongation of 120°
   (*b*) to be in quadrature
   (*c*) to be gibbous
   (*d*) to be a crescent
   (*e*) to be half-full
   (*f*) to rise only in the morning or evening:

State, in each case, whether it is a superior or an inferior planet. Give reasons for your answers.

3. Find the apparent breadth of the visible portion of a planet, when its elongation is 30° and the ratio of its distance from the sun to that of the earth from the sun, $\sin^{-1}\frac{1}{2}$.

4. Find the apparent breadth of the phase, when the angle subtended at the planet by SE is 60°.

5. The synodic period of Jupiter is 399 days. Find its sidereal period.

6. The sidereal period of Venus is 224 days; find the interval between its successive inferior conjunctions.

[1] With the Lick telescope, Barnard has been able to follow a comet so far on its outward journey, as to raise hopes that we may perhaps soon be able to follow some comet all round its orbit. [Turner.]

7. Verify Kepler's third law from the following data :—

(*a*) Periodic time of mercury = 88 days.

Its mean distance = ·38 times the Earth's distance from the sun.

(*b*) Synodic period of Mars = 780 days.

Its mean distance = 1·52 earth's distance.

8. Assuming Kepler's third law, find the velocity of Mercury, in its orbit, given the mean distance of Mercury from the sun = ·38 of the earth's distance.

9. Find the ratio of the distances from the primary of two of the satellites of Jupiter (Europa and Ganymede), given that their periodic times are in the ratio of 1 : 2. If the distance of the first is 9,400 miles, find that of the other.

10. If the velocity of Mars in its orbit is 15 miles per second, find the velocity of Saturn, assuming Bode's law.

11. Explain why the planets and comets have their maximum velocity at perihelion.

How does the angular velocity of a planet change with distance ?

12. Explain how you would prove that the earth's path about the sun is an ellipse.

The greatest and least angular semi-diameters of the sun are observed. Show how with the help of these data, the earth's path can be described, geometrically.

13. Explain how it has been concluded that the earth is a planet, obeying Kepler's laws.

# CHAPTER VIII

## Position of the first point of Aries

### *Precession and Nutation*

1. We have seen that the R.A. of a star can be determined with the help of a transit instrument and an astronomical clock, so set as to indicate $0^h\ 0^m\ 0^s$, when the first point of Aries is in the meridian of the place of observation and that the clock can be so set, if we know the R.A. of one star, *independently* of *this operation.*

2. The two problems (*viz.*, that of setting the clock and that of determining the R.A. of a star, independently of this are in fact, as we shall presently see, identical.

3. Let $\sigma$ (fig. 70) be a star and $\sigma$L, its declination circle, intersecting the equator at L.

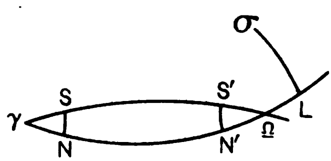


Fig. 70.

Observe the *difference* of R.A. of the Sun and the star, whose R.A. has to be determined, shortly after the passage of the former through vernal equinox. Let this be $a$. This being the *difference* in the sidereal time of the transits of the sun and the star, at the moment, it can be determined by means of an astronomical clock, although the clock may not yet have been set, so as to indicate sidereal time *absolutely*.

Let the declination $\delta$ of the sun be determined at the same time ; *viz.*, SN$=\delta$. (SN being the declination circle

of the sun, intersecting the equator at N, at the time of observation.)

Observe, next, the difference in R.A. between the sun and the star, when the declination of the sun is again equal to $\delta$, *i.e.*, shortly before autumnal equinox.

Let this be $\beta$ and let $S'N'$ be the declination circle of the sun ($S'$) at the second observation, intersecting the equator at $N'$.

$$\text{Then } \gamma L-\gamma N=\alpha$$

$$\text{and } \gamma L-\gamma N'=\beta.$$

$$\therefore\ 2\gamma L-(\gamma N+\gamma N')=\alpha+\beta\,;$$

$$\text{also } \Omega N'=\gamma\ N=180-\gamma N'.$$

$$\therefore\ 2\gamma L-180^\circ=\alpha+\beta.$$

$$\therefore\ \gamma L=\frac{\alpha+\beta+180^\circ}{2}=\text{R.A. of the star.}$$

4. Hence, the R.A. of the star is known, independently of an astronomical clock, *properly* set. The clock may now be set, so that it may indicate the R A. of the star, when it is crossing the meridian. Then, also, it will indicate $0^h$, $0^m$, $0^s$, when the first point of aries is in the meridian. In other words, $\gamma$ is the point on the equator which is in the meridian, when the clock indicates $0^h$, $0^m$, $0^s$. Thus, at the same time, the position of the first point of Aries is determined.

5. When the first point of Aries has been determined, with due care, it is found that it has a motion along the ecliptic at the rate of $50''{\cdot}24$, per year, in a direction, opposite to the sun's annual motion.

This is known as the **precession of equinoxes.**

6. The intersection of the equator with the ecliptic is called the line of equinoxes. This line rotates about the pole of the ecliptic, on account of precession.

On account of this, the R.A. of a star will be slightly different at the second observation from what it is at the first.[1]

If the displacement of $\gamma$ in the interval is $\epsilon$, then we have $\gamma L = \frac{180+\alpha+\beta-\epsilon}{2}$

7. In the above, we have postulated that the second observation is made, when the declination of the sun is the same, as at the first observation. In practice, it will not be possible to note the exact instant, at which this is accurately the case. In order to obviate this difficulty, the following device is resorted to :—Observation is made when the declination of the sun is $\delta_1$, slightly in excess of $\delta$, (where the declination S′ N′ is equal to $\delta$) and again, when the declination is slightly less than SN, say, $\delta_2$.

Then, during the interval, the change in the declination may be assumed to be proportional to the change of R.A. of the sun and, thus, the moment, when the declination would be just equal to $\delta$ may be calculated, as well as the corresponding difference of R.A.

[1] When the path of the sun on the celestial sphere had been accurately traced, it must have been soon apparent that the sun did not come back to the same point of a sign, at succeeding equinoxes; in other words, the line of equinoxes points to different stars at different times, having a retrograde motion and completing a cycle in about 26,000 years. This is known as the precession of equinoxes. Hipparcus (134 B.C.) was led to its discovery, by comparing his own determination of the longitude (*i.e.*, angular distance from the first point of Aries, along the ecliptic) of certain stars with those of Teniocharis, about 150 years earlier but the fact of precession must have been surmised quite early in the history of astronomy. It is in fact, maintained by some that the Hindu Astronomers knew and had determined its rate before 1192 B.C.—necessarily roughly. Their determination (whatever may have been its date—it is mentioned in *Suryya Siddhanta*) was more accurate than that of Ptolemy, as they supposed the displacement in a century to be 1⅓°.

Thus, let $S_1N_1=\delta_1$,

$S_2N_2=\delta_2$, where $S_1$, $S_2$ are the two positions of the sun on either side of $S'$ and $S_1N_1$, $S_2N_2$, the corresponding declinations.

Then, $\frac{N_1N'}{N_1N_2}=\frac{\delta_1-\delta}{\delta_1-\delta_2}$; or, $N_1N'=\frac{\delta_1-\delta}{\delta_1-\delta_2}N_1N_2$.

Hence, if $N_1L=\beta_1$, $N'L=\beta$, $N_2L=\beta_2$,

then, $N_1N_2=\beta_1-\beta_2$ and $\beta=\beta_1-N_1N'$

$\therefore\ \beta=\beta_1+\frac{\delta_1-\delta}{\delta_2-\delta_1}(\beta_1-\beta_2)$.

8. The method is due to Flamsteed and has the advantage that it requires, not the absolute declinations but changes in the declinations, so that all uncertainties as to the value of latitude of the place of observation (a knowledge of which is required for the determination of declination) or instrumental and other errors will not affect the result.

9. *Obs.* In order to determine the position of the ecliptic, at any time, we must know its points of intersection with the equator, at that time, and also its inclination to the equator, called the **obliquity** of the ecliptic

## Obliquity of the Ecliptic.

The obliquity of the ecliptic is equal to the declination of the sun at the solstices.

Now, we know that

the latitude = zenith distance—declination.

If we call this $l$,

$l=z+\omega$ at the summer solstice,

for, then, the declination of the sun $=\omega$.

also, $l=z'-\omega$, at the winter solstice.

$\therefore\ \omega=\frac{z'-z}{2}$, where $\omega$ is the obliquity and $z$ and $z'$, zenith distances, at the summer and the winter solstice, respectively.

We have seen that the declination of the sun changes very slowly at the solstices. It is not therefore, possible to make observation at the exact instance at which the declination is maximum. Resort has in fact to be made to a method similar to 'Flamsteed's method.' [Art. 8.]

10. Besides the three systems of co-ordinates already discussed, a fourth system of co-ordinates are in use, referred to the ecliptic and its secondary through $\gamma$ and $\Omega$.

*Def.* The distance of a body from the ecliptic, measured along the secondary to the ecliptic through the body is called the **Celestial Latitude.**

The **Celestial Longitude.** of a body is the arc of the ecliptic intercepted between the first point of Aries and the secondary of the ecliptic through the body.

11. We have seen that the first point of Aries has a retrograde motion along the ecliptic, at the rate of $50''{\cdot}25$ annually.

It is, moreover, found that the celestial latitudes of stars are practically constant. Hence, since stars are bodies which are practically fixed in space, we conclude that the ecliptic is a practically fixed plane in space.

If we represent the ecliptic on the celestial sphere (referred to the centre of the earth) by a fixed great circle, then K the pole of the ecliptic must be also fixed. Let, now, $P_0$ be the pole of the equator, O, the centre of the celestial sphere and $\gamma$, the first point of Aries (as usual).

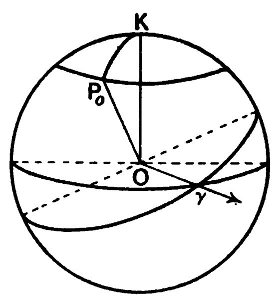


Fig. 70. A

Then $O\gamma$ is the line of intersection of the equator and the ecliptic and is therefore perpendicular to both $OP_0$

the earth's axis and OK, the normal to the ecliptic. Thus, $O\gamma$ is perpendicular to the plane $OKP_0$.

Now, since $\gamma$ moves along the ecliptic, with uniform angular velocity, the plane $OP_0K$ also must move with uniform angular velocity about OK. In other words, $P_0$ must describe uniformly a small circle about K, since the obliquity of the ecliptic (that is, the angle $P_0OK$) is constant.

The motion of $OP_0$ about OK is, thus, one of steady rotation of the same kind as that of the axis of a top (fig. 71) spinning steadily about an axis, inclined to the vertical. Now, since the earth has a motion of rotation about the polar axis, the analogy between the two motions may be regarded as complete:

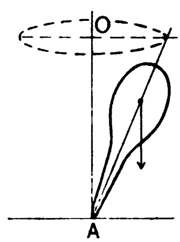



Fig. 71.

12. In the case of the spinning top, there is a couple acting, due to the force of gravity, downwards and the reaction of the ground, upwards. If this couple had been operative on a stationary top, the top would have fallen away from the vertical, the axis, tending to come into coincidence with the horizontal plane on which the top rests. As the top is spinning, however, this spin combines with that generated by the acting couple, in each element of time, so as to produce a displacement which carries the axis of the top round the vertical. This motion is called **precessional** motion.

13. In the corresponding problem of the earth, the couple is due to the action of the sun and the moon and, to a very small extent, that of the planets on the protuberant portion of the earth, tending to move the earth's axis, towards the ecliptic.

14. *Def.* **Solstitial colure** is the great circle perpendicular to the line of equinoxes. It rotates about OK and completes a cycle in about 26,000 years.

15. The discovery of precession was essential to all accurate astronomical calculations, especially in the matter of the calendar and much confusion arose in the past and still exists in the calendars of various nations, as sufficient account has not, in many cases, been taken of this factor. For, if we define in general terms, the year (say the solar year) as the period in which the sun completes a cycle, it is easily seen that the cycle may be taken to refer to any fixed point in space, back to the same point or from an equinox to the same equinox. The first is called the sidereal year; the second, the tropical year, the difference in the two being due mainly to precession. As the second marks the recurrence of the seasons, it, alone, is useful as a practical unit, so that any other mode of reckoning will naturally create confusion, for in that case, a particular point of time in the year will not always correspond to a particular season (so far, at any rate, as it is determined by the sun's motion).

There is a further significance attached to the phenomena of precession, from a historical point of view. For, as we have seen, one effect of precession, is that the axis of the earth points to different points on the celestial vault, lying [but for *nutation* (art. 16)] on a small circle. All those stars, therefore, that lie on or near that circle must be *pole stars* to the earth, in succession. In the same way, the equinoctial points, also, come to be occupied

by different stars. Such stars are, naturally, for the time being, celestial bodies of special importance. Accordingly, if any star is specially mentioned in the writings of any period, we may reasonably conclude that this period refers to the epoch, at which the star is a polar or an equinoctial star. Similarly, if a particular direction is found to be signalised, in any way, in any architecture, as in the pyramids of Egypt, we shall not be far wrong, if we identify it as the direction of the polar axis or the line of equinoxes, at the epoch, considered. Considerations like these have been variously helpful to the historian and the archæologist. We have already adverted to the evidence it affords of the antiquity of the Hindu system of lunar zodiac. [Introduction.]

16. Now, we have assumed, so far, that the rotation of the earth's axis, about the axis of (or the normal to) the ecliptic is steady,[1] that is the obliquity of the ecliptic (or the arc $P_0K$) is a constant quantity.

Observation however, shows, that this is not, altogether, the case. The inclination, in fact, is found to suffer periodic variations within narrow limits.

This, indeed, would be *a priori* evident. The effect of the action of the sun and the moon, to which the motion of the point $P_0$ is due undergoes (as a little consideration will show) small periodic changes, the effect of which is that $P_0$ suffers a disturbance, which has small periodic components along and perpendicular to $KP_0$. It follows

[1] To fix our ideas, imagine a circle drawn through *Polaris*, *Draconis* and *Hercules*, with its centre, at the pole of the ecliptic, that is, a point about $23\frac{1}{2}^{\circ}$ from Polaris; then, if we describe a cone with this circle as base and the observer's eye as the apex, the motion of the axis of the earth may be described as sweeping out this cone in about 26,000 years. It follows, therefore, that 'Hercules' will be a pole star at some period or other (about 15000 A.D.); similarly, Draconis was a pole star about 3000 B.C., *i.e.*, at, about, the age of the pyramids of Egypt.

accordingly that $P_0$ is not the actual pole. If P be the actual (north) pole of the earth then the motion of P is found, in fact, to be made up as follows :

Imagine a point $P_0$ to move about K with the steady precessional motion of $50''\cdot25$, a year. Let a point P have, at each moment, a small periodic motion along and perpendicular to $KP_0$, relatively to $P_0$; then P will be the pole of ecliptic.

The small periodic motion along $KP_0$ (which produces a change in the obliquity) and that, perpendicular to $KP_0$ constitute **Nutation**.

Again we have proceeded on the assumption that K, the pole of the ecliptic is fixed. This is not actually the case. Its change of position is, however, slight. This is due to the disturbing action of the planets, the effect of which is to produce a slight change in the position of the ecliptic.

EXERCISE.

1. The sidereal interval between the transits of a star and the sun when the declination of the sun is 6° is 15 hours, on a certain day, and it is equal to 10 hours, when the sun has the same declination, again. Find the R.A. of the star, as well as those of the sun, at these two epochs.

2. Given the declination of the sun on a certain day, show how to describe the ecliptic.

3. Explain why on account of precession, the intervals between the passages of the meridian through the same star differ from a mean sidereal day.

4. What is the present longitude of a star which was the pole star in 15 B.C.?

5. Which plane attached to the spinning top corresponds to the plane of solstitial colure ?

## CHAPTER IX

### Astronomical Refraction

1. Since a celestial body is seen by means of the rays of light proceeding from it to the eye, any deviation that these rays undergo must produce an apparent displacement in the position of the body. Accordingly, this must be allowed for, in order that the results obtained may be free from the extraneous source of error, to which it gives rise.

2. As the light, coming from a heavenly body has to pass through the earth's atmosphere, consisting of layers of air of increasing density, it is refracted or bent more and more towards the vertical, so that, on reaching the eye, it appears to come from a point, whose altitude is greater than that of the object from which it comes. The object, accordingly, appears to be raised above its true position.

3. Proceeding to details, it is necessary to remember that from considerations of symmetry, the atmosphere may be assumed to consist of concentric layers of equal density—if we regard the earth to be spherical.

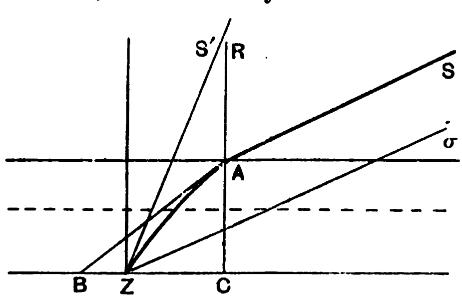


Fig. 72.

Also, the laws of refraction are (1) that the sine of the angle of incidence is proportional to the sine of the angl of refraction and (2) the incident ray, the refracted ray and the normal to the surface of separation between consecutive atmospheric layers lie in the same plane.

Thus, a ray of light from S, say SA, on reaching the atmosphere will be refracted along AB (fig. 72) such that $\frac{\sin SAR}{\sin BAC}=\mu$, where $\mu$ is the index of refraction of the uppermost layer of the atmosphere, and depends on the density of this layer and RAC is the direction of the normal to the surface of this (uppermost) layer, supposed to be of uniform density. (From symmetry, RAC may be taken to pass through the centre of the earth.)

There will be refraction again at the next layer and so on, till the ray reaches the eye at Z (in the direction of ZS′ say).

Accordingly, the apparent direction of the star will be ZS′, instead of $Z\sigma$ (where $Z\sigma$ in parallel to AS).

Moreover, since the lines AS, BA and RA are in the same plane (the earth being regarded as spherical), we observe that from the second law of refraction, the vertical plane in which the star is actually seen is the same as that in which it would be seen, if there were no refraction.

Hence, it follows that on account of refraction, the *azimuth of a star does not change but the zenith distance is decreased.*

4. In order to find, approximately, the amount of deviation that a ray undergoes, we shall assume the atmosphere to be plane and to be replaced by a single layer, which will produce the same deviation of the ray, as the actual atmosphere does.

Then if $z'$ = the apparent zenith distance (fig. 73) and $r$ = the refraction, *i.e.*, the deviation of the ray on account of refraction, we have

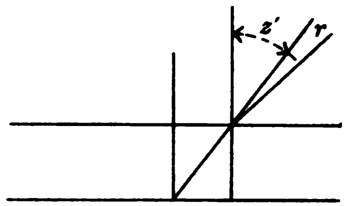


Fig. 73.

$$\frac{\sin (z'+r)}{\sin z'} = \mu$$ where $\mu$ is the index of refraction, of the single layer by which we have replaced the actual atmosphere.

Since $r$ is small, we have

$$\sin z' + r \cos z' = \mu \sin z', \text{ nearly.}$$

$$\text{Hence, } r = (\mu - 1) \tan z' \text{ nearly.}$$

Here, $\mu$ depends on the *density* of the equivalent layer of the atmosphere; that is, the *mean density* of the atmosphere and this, again, depends on various atmospheric conditions, especially *temperature* and *pressure*. Within certain limits, however, we may take this to be constant $(=k+1)$, so that the law of refraction is given by the simple formula,

$$r = k \tan z'$$

(where $z'$ is the apparent zenith distance).

5. As $\mu$ is the index of refraction of an imaginary atmosphere, which produces the same effect as the actual atmosphere, the constant $k$ is best determined by direct observation.[1]

6. The principle of the method of observation is as follows :—

We observe the zenith distance of a body at a position A and again at a position B. Then, from the observed or

[1] In order to obtain it by calculation, it will be necessary to know the manner in which the density of the atmosphere varies and the exact relation between the index of refraction and density. These are difficult to ascertain and are rather uncertain.

*apparent* zenith distances, we can deduce the real zenith distances, on the assumption of the above law. If, therefore, we know (on theoretical grounds), what the relation between the real zenith distances at A and B should be, we have an equation to determine $k$.

7. In order to apply this method, in practice, observe the meridian zenith distance of a circumpolar star at upper and lower culminations at $\sigma_1$ and $\sigma_2$ (not marked on the diagram). Let these be $z_1$ and $z_2$.

Then, their real zenith distances must be

$$z_1 + k \tan z_1 \quad \text{and} \quad z_2 + k \tan z_2.$$

But we know that the sum of their real zenith distances

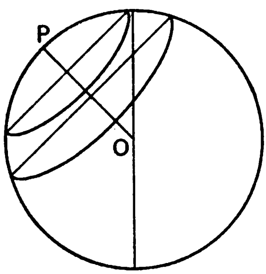



Fig. 73.

$$= Z\sigma_1 + Z\sigma_2 = PZ - P\sigma_1 + PZ + P\sigma_2$$

$$= 2\ PZ, \text{ since } P\sigma_1 = P\sigma_2$$

$$= 2 \text{ colatitude.}$$

$$\therefore\ 2 \text{ colat} = z_1 + z_2 + k (\tan z_1 + \tan z_2).$$

Hence $k$ is known since $z_1$ and $z_2$ have been observed, the latitude of the place being supposed known.

If the latitude is not known, we have to make another pair of observations, say, the zenith distances, $z_1'$ and $z_2'$ of another circumpolar star. This gives

$$2 \text{ colat} = z_1 + z_2 + k (\tan z_1 + \tan z_2)$$

$$= z_1' + z_2' + k (\tan z_1' + \tan z_2');$$

$$\text{whence, } k = \frac{(z_1 + z_2) - (z_1' + z_2')}{\tan z_1' + \tan z_2' - (\tan z_1 + \tan z_2)}.$$

8. In actual practice, it is more convenient to observe the zenith distances of the sun at summer and winter solstice, say, $s_1$, and $s_2$.

Then, if $s$ and $s'$ are the true zenith distances of the sun, at these epochs, we have

$$s+\delta=s'-\delta=\text{latitude},$$

where $\delta$ is the declination of the sun at either solstice. Hence,

$$2\ \text{lat}=s+s'=s_1+s_2+k\ (\tan s_1+\tan s_2).$$

If, finally, latitude is unknown, it can be eliminated by observing a circumpolar star, as in art. 7.

9. The effect of refraction can be represented on a diagram, as follows :—

Let $S_0$ be the position of a star (fig. 74). Then since refraction evidently increases the altitude without changing the azimuth, S will be the apparent position of the body, where $ZSS_0$ is the *vertical* of the star.

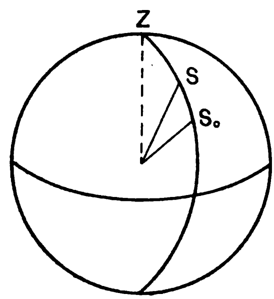


Fig. 74.

10. **The effects of refraction.**

(*1*) *On the time of sun-rise and sun-set.*—At the horizon, where $z=90°$, the above formula obviously fails. From actual observation, however, it has been found that the *horizontal* refraction ranges between 34′ to 39′. As this quantity is slightly greater than the diameter of the sun, it follows that when the sun's lower limb appears to be rising, its upper limb may still be below the horizon. Thus, the time of sun-rise is accelerated and, similarly, that of sun-set, delayed, on account of refraction. The effects in the case of the moon are similar.

(2) *On the form and size of the solar and lunar discs.*—In order to show this, let us represent, on the celestial sphere, the undistorted disc of the sun in the first place. Let this be AB (fig. 75). Let ZAB be the vertical circle and CD a parallel to the horizon through the centre of the disc.

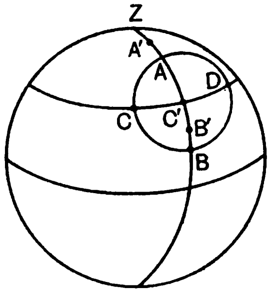



Fig. 75.

Then A is displaced to a point A′ above A in the vertical circle ZAB and B to a point B′.

But since the zenith distance of B is greater than that of A, BB′ is greater than AA′. Hence A′B′ is less than AB. On the other hand, the diameter CD remains practically unchanged. The result is that the circular disc of the sun becomes an ellipse, with its minor axis along the vertical through its centre.

11. The formulæ for refraction can of course, be expressed in terms of apparent altitude, or real zenith distance.

Thus, since $r=k \tan z'$,

where $z'$ is the apparent zenith distance,
we have $z-z'=k \tan z'$,
where $z$ is the true zenith distance.

Since $z=90°-a$, $z'=90°-a'$ where $a$ and $a'$ are the corresponding altitudes, we have $a'-a=k \cot a'$

Again, $r=k \tan z'=k \tan (z-r)$

$$=\frac{k (\tan z-r)}{1+r \tan z}$$

$\therefore\ r=k \tan z$, if $r^2$, $kr$ (both of the second order) are neglected.

### TWILIGHT.

12. All these phenomena are due to refraction of light. But light also suffers partial reflection, at the successive layers of the atmosphere and the solid particles suspended in it. The most striking phenomenon to which such reflections give rise is that of **twilight**. This is caused by irregular reflection of solar light, at the upper regions of the atmosphere, when the sun is actually below the horizon. The effect of this is that, at sun-set, complete darkness does not set in, at once, nor is it completely light at sun-rise but that there is a period of varying light between the moment of actual sun-set or sun-rise and that of complete darkness. It has been found that at the moment of complete darkness, the sun's zenith distance is on an average 108°.

13. Accordingly, the duration of twilight will necessarily be different at different places and in different seasons. In order to determine this, it is necessary to find the position of the sun at any time at any place, when this zenith distance is 108°:

Describe the celestial sphere of the place and trace the diurnal path of the sun, from his known declination (which is a parallel to the celestial equator at a distance, equal to the declination, *viz.*, SS′, fig. 76).

Describe, also, a vertical circle (*i.e.*, a great circle passing through the zenith and nadir), such that ZS is equal to 108° Then, S will be the required position of the sun, at the moment when twilight ends (or begins) at the place, considered.

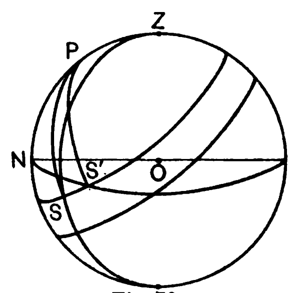


Fig. 76.

If now S′ is the position of sun-rise or sun-set, the difference in the hour-angle of S and S′ (*viz.* $\angle$ SPS′) gives the duration of twilight. It is clear, therefore, that the duration of twilight will depend on the place of observation and the declination of the sun.

14. At the equator, P (the North celestial pole) coincides with the North point (N). Here, at the equinoxes, the corresponding difference in the hour angle is 18° or $\frac{18}{15}$ hours, showing that, at the equinoxes, the duration of twilight is $1^h\ 12^m$ at the equator.

15. In order that twilight may last all night, it should end at midnight and therefore, the point S defining the position of the sun, when twilight ends should be on the celestial meridian, below the horizon. In other words, the distance of N from the sun's diurnal circle should be equal to 18°. That is, the colatitude should be equal to 18° + $\delta$ (where $\delta$ is the declination) ;

$$\text{or } 90° - l = 18° + \delta$$

$$\text{or } 72° = l + \delta \text{ (where } l \text{ is the latitude).}$$

At the poles, there will be a long period of continuous twilight.

EXERCISE.

1. Find the latitude of the place at which twilight just lasts all night, when the sun's declination is 20°N. [Ans. $l + 20° = 72°$.]

2. Find the declination of the sun, when twilight just lasts all night at the latitude of 55°. [17°.]

3. Is it possible for a place in latitude 25° to get twilight all night ? [No.]

4. What is the lowest latitude which can have twilight, the whole night. [72°—23°28′.]

5. Explain by means of a diagram what will be the duration of twilight, at the equator at the solstices and also at the poles,

## CHAPTER X

### ABERRATION

1. In the case of refraction, a ray of light from an object undergoes actual deflection, in passing through the atmosphere, thus producing an apparent displacement of the object.

2. Since, however, the observer is in motion (on account of the earth's diurnal motion), the direction in which a star is seen is the direction in which light *appears* to come, while if the earth were at rest, a star would be seen by means of light, issuing from it and coming to the observer with a known velocity, in the direction in which light actually proceeds from the star. The earth's motion produces an apparent displacement, called **aberration.**

3. Thus, if E (fig. 77) is the position of the earth (*i. e.*, the observer), $\sigma$ that of a star in space (actual), then, $E\sigma$ is the direction in which the star would be seen, if the earth were at rest. But since the earth is in motion in its orbit, the direction in which a star is seen will be different.

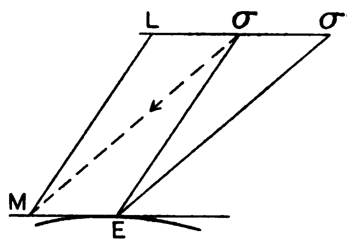


Fig. 77.

We proceed to consider what this apparent direction will be.

4. To simplify the explanation, we shall assume, as on the corpuscular theory of light, that something in the nature of a material particle is emitted from the star, which

moving with the velocity of light, produces, on reaching the observer's eye, the sensation of vision.

Now, in order to find the direction in which this reaches the eye, we must superpose on the system consisting of bodies E and $\sigma$, a velocity equal and opposite to that of E.

Let $\sigma$L (fig. 77) represent the velocity of the Earth on the same scale, on which $\sigma$E represents the velocity of light.

Then, completing the parallelogram, we get the direction ($\sigma$M), in which *light appears to come,* on the principle of relative velocities; or if we draw E$\sigma'$ parallel to M$\sigma$ and produce L$\sigma'$ to meet E$\sigma'$ at $\sigma'$, $\sigma'$ will be the position of the star, as seen by the observer, just in the same way, as, to a man walking forwards, a rain drop falling vertically appears to come towards him in a slanting direction.

This displacement ($\sigma\sigma'$) is called aberration.

5. We may follow out the consequences, further. Let EA be the earth's orbit, E, $\sigma$, the position of the earth and the star in space, $\sigma'$ being the displaced position of the star (fig. 78). Also let AB be the ecliptic, K its pole

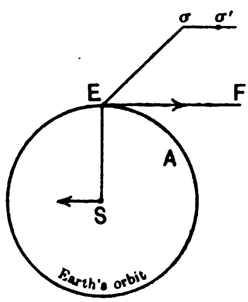



Fig. 78.

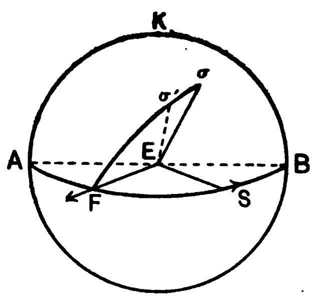



Fig. 79.

$\sigma$, a star, S the sun, E the earth (at the centre of the celestial sphere) (fig. 79). Then if we draw EF perpendicular to ES (*in both figures*), EF will be the direction of motion of the earth, assuming its orbit to be circular.

[In fig. 78, the plane of the paper coincides with the plane of the ecliptic, while in the second figure (fig. 79), it is perpendicular to that plane.]

Also, the angle $\sigma$EF represents the same angle in both diagrams (figs. 78 and 79).

Then, on the celestial sphere also, $\sigma$ is the real position of the star and $\sigma'$, the displaced position on account of aberration. The displacement, accordingly takes place along a great circle through $\sigma$ and F, where F is a point on the ecliptic at a distance of 90° from S, since SE is at right angles to EF (fig. 79).

6. Again, since EF is the direction of motion of the earth, the apparent direction of motion of S is from S to B as in the figure 79, which proves that the point F is *behind* the sun. Finally, from the triangle $\sigma E \sigma'$ (fig. 77).

$$\frac{\sigma\sigma'}{\sigma E} = \frac{\text{velocity of the earth}}{\text{velocity of light}} = \frac{\sin \sigma E \sigma'}{\sin \sigma\sigma' E}.$$

Thus, $K \sin \sigma\sigma' E = \sin \sigma E \sigma'$

*i.e.*, $\sin \sigma\sigma' = K \sin \sigma' F$ (fig. 79),

if we put the ratio of the velocity of the earth to the velocity of light $=K$, a small constant.

Moreover, $K \sin \sigma' F = K \sin \sigma F$ nearly, since K and the angle $\sigma E \sigma'$ are both small. [In fact $K = 20''\cdot 49$.]

Since $\sigma\sigma'$ is small, we get

$$\sigma\sigma' = K \sin \sigma F, \text{ nearly.}$$

7. We conclude, accordingly, that the star $\sigma$ aberrates towards a point F, 90° behind the sun and the aberration $\sigma\sigma'$ is equal to $K \sin \sigma F$.

The angle $\sigma$EF is called the earth's way. Thus, the law may be stated thus :—

**aberration = K sin of earth's way.**

8. As the sun moves in its orbit, $\sigma$ will follow the sun, completing a cycle in one year. Moreover, the minimum

value of $\sigma F$ is $\beta$, if $\beta$ is the arc of the great circle through the star perp. to the ecliptic.

(For from a point on a sphere, the shortest arc of a great circle that can be drawn to another great circle, is the intercept of the great circle, perp. to the latter.)

But $\beta$ is the celestial latitude of the star.

Hence, the minimum value of $\sigma\sigma'$

$= K$ sin (celestial latitude).

The locus of $\sigma'$, on the celestial sphere is, therefore, a small curvilinear ellipse, with centre $\sigma$, and having the following dimensions :—

major axis $= K$ ;

minor axis $= K$ sin (celestial latitude of the star).

9. As to the path of the star in space, due to aberration, we observe that this is the locus of $\sigma'$ (fig. 77). Now $\sigma\sigma'$, is proportional to the velocity of the earth and is parallel to this velocity. The locus of $\sigma'$ is, therefore, the *hodograph* of the earth's orbit, with reference to the real position of the star, taken as pole.

This is known to be a circle, and its plain is evidently parallel to the earth's orbit.

10. It is worthy of note, moreover, that the displacement curve of a star on the celestial sphere is an element of the ecliptic—practically, a small portion of a straight line, if the star is on the ecliptic. For, in this case, the cel. latitude is zero.

11. Also, when the star is at the pole of the ecliptic, the path becomes a circle ; for then, the cel. lat. being 90°, its minor axis is equal to the major axis.

12. All these are facts of observation, so that actual calculations based on the principle of aberration account completely for the displacements observed. In fact, the observed fact that all stars partake of a common motion, describing the hodograph of the earth's orbit in a plane

parallel to that orbit, in the *same* period (*viz.*, one year) is evidence, the cogency of which, it is impossible to resist, leading to the conclusion that this motion is apparent motion and can only be due to the motion of the observer, carried by the earth in its motion round the sun. The phenomenon of aberration, thus, supplies an *ocular* demonstration of the earth's annual motion round the sun.

## EXERCISE.

1. The apparent meridian altitudes of a circumpolar star are 25° and 30°. Find the latitude of the place, given the co-efficient of refraction = 58″·2.

2. The apparent altitude of a star is $\sin^{-1}\frac{5}{13}$ find its true altitude.

3. The apparent zenith distances of two circumpolar stars at their meridian passage are (1) 30° and 40°, (2) 25°, 45°, find the latitude of the place.

4. If the apparent meridian zenith distance of the sun at summer and winter solstice are 30° and 40°, find the latitude of the place.

5. Represent on the celestial sphere, the effect of refraction on the distance between two stars.

6. Explain why the azimuth of a star is unaltered by refraction. Would the shape of the earth have any effect on the phenomenon?

7. Show, by means of a diagram, the effect of refraction on the rising and setting of a celestial body.

8. Compare the effect of refraction and aberration in their geometrical aspects.

9. If the velocity of the earth is doubled, what is the effect in the nature of the aberrational ellipse of a star.

10. Explain how the dimensions of the orbits described by stars in space about their actual positions differ from each other. Are they all similar?

11. Indicate on the celestial sphere, the position of a star, if any, such that the effect of aberration is annulled by the effect of refraction at a given moment, at a given place.

12. Indicate on the celestial sphere, the position of a star, if any, such that its declination is unaffected by aberration, at a given moment.

If $\sigma$ be the star and F is the point on the ecliptic to which stars aberrate, and P is the pole of the equator, then $\sigma$P should be perpendicular to $\sigma$F.

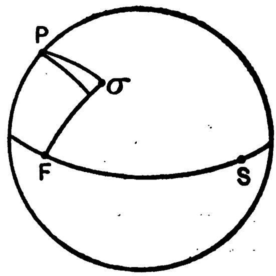



Fig. 80.

## CHAPTER XI

### Geocentric Parallax

1. We have seen that the direction in which a star is seen remains the same, no matter how we change our position on the surface of the earth. This is due to the fact that stars are so far off that the angle subtended at any, the nearest of them, by the greatest distance that we can measure on the earth's surface (*viz.*, the earth's diameter) is absolutely negligible.

2. The distances of the members of the solar system, however, being of much lesser magnitude, the angles subtended at these bodies by the diameter of the earth, though small, are still measurable.

Taking the case of our nearest neighbour, the moon, it has been found from observation that its distance from the earth varies (in round numbers) between 253,000 miles and 221,600 miles. The method of measuring this distance is practically the same as would be used for measuring the distance of a very distant object on the surface of the earth. For this, we measure a suitable length on the ground and find the angle subtended at the object by this *base line*. Then, if the distance is so great and the angle, so small, that the triangle (formed by the object and the assumed base line) can be taken to be isosceles, then the distance of the object is about 3,500 times the base, if the angle is one minute.

In the case of the moon, the base line has to be proportionately longer and the measurement of the angle, on account of its minuteness, has to be conducted by indirect astronomical means.

3. *Def.* The angle subtended at a celestial body by the radius of the earth is called its *geocentric parallax.*

When the body is at the horizon, the corresponding geocentric parallax is called **horizontal parallax.**

4. Assuming the earth to be a sphere, it easily follows that if $p$ is the horizontal parallax,

$$\sin p = \frac{r}{D},$$

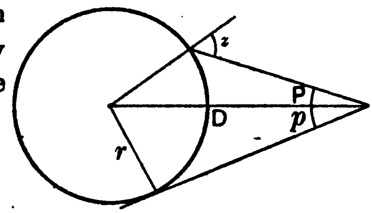



Fig. 81.

where $r$ = the radius of the earth and D = distance of the body from the centre of the earth.

Also if P = the geocentric parallax (fig. 81) *generally*, then $\dfrac{\sin P}{\sin z} = \dfrac{r}{D}$ where $z$ is the zenith distance of the body, as seen by an observer on the surface of the earth. This is called apparent zenith distance.

5. Now, since the quantities $p$ and P are small, we may take

$$\sin P = P, \sin p = p$$

$$\text{and therefore } P = p \sin z,$$

the angles being expressed in circular measure.

6. Before proceeding to discuss the methods of measuring these quantities, in the case of the various members of the solar system, we shall briefly consider its importance in Astronomy.

7. From the equation $p = \dfrac{r}{D}$, it easily follows that if we know $p$, we can find D, or the distance of the body from the centre of the earth, the radius of the earth being known.

Thus, since, in the case of the moon the greatest and least values of the horizontal parallax are, $61'27''$ and $53'\ 54''$ we conclude that the greatest distance of the moon $=$ 252,658 miles and the least distance of the moon $=$ 221,616 miles. Similarly, since the mean parallax of the sun is $8''\cdot8$, its distance is nearly 92,852,000 miles. (earth's rad. $=$ 3,963 miles).

8. We can, also, refer the positions of bodies to the centre of the earth, if we know their parallax.

We have seen that the position of a body on the celestial sphere of an observer is necessarily *its angular position*, and this is also the same as its angular position in space, *as seen by the observer*. In the case of infinitely distant bodies, like stars, the variation in the angular position is absolutely negligible, no matter how the position of the observer changes on the surface of the earth but for bodies of the solar system, this variation—due to a change in the position of the observer—is appreciable, though small.

9. In order, therefore, that observations made at different places should be comparable, it is necessary to reduce all observations to a standard position—common to all observers. This standard position is, evidently, the centre of the earth.

This reduction requires a knowledge of parallax.

For, if the zenith distance of a star as seen by an observer at any place is $z$, then, evidently,

$$z = z_0 + \mathrm{P}.$$

when $z_0$ = zenith distance as seen from the centre of the earth, $z_0$, being measured on the supposition that the vertical to the observer, when transferred to the centre of the earth, is the same as the vertical of the observer at the surface.

$z_0$ is called the *true zenith* distance and $z$, the *apparent* zenith distance.

10. The position of a body on the celestial sphere, corresponding to the true zenith distance is called the true position of the body, that corresponding to apparent zenith distance, being called the apparent position.

11. If we assume the earth to be spherical, the direction of the normal at any point on the surface passes through the centre.

In this case, therefore, the azimuth of the body is the same whether the position be referred to the centre or to the observer at the surface.

12. The relative positions can, thus, be represented very simply on a diagram. For if M (fig. 82) be the position of a celestial body referred to the observer's celestial sphere at a point on the surface of the earth, $M_0$ will be the corresponding position referred to the centre, where $M_0M$ is on the vertical through M and will be *above* M, since $z_0$ is less than $z$. Also, $M_0M = P$.

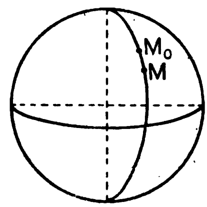



Fig. 82.

Note that the above change in the position of the body will affect *both* its R.A. and declination.

## Lunar parallax.

13. Let A, B be two places in the same meridian, very far apart, one in the Northern and the other in the Southern Hemisphere. [Fig. 83.]

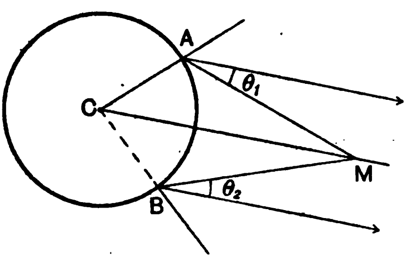



Fig. 83.

Let $z_1$ = apparent zenith distance at A,

$z_2$ = apparent zenith distance at B,

$P_1$ = parallax at A.
$= \angle AMC$,

and $P_2 = \angle BMC$.

Then,

$$P_1 = p \sin z_1$$
$$P_2 = p \sin z_2$$
$$\therefore \; P_1 + P_2 = p(\sin z_1 + \sin z_2)$$
$$= \text{the circular measure of } \angle AMB$$
$$\therefore \; p = \frac{\theta}{\sin z_1 + \sin z_2},$$

if the circular measure of $\angle AMB = \theta$.

$\therefore$ if $z_1$, $z_2$ are observed at the two stations and the angle AMB is determined, we can find the horizontal parallax.

14. In order to determine the $\angle$AMB, we may proceed in one of the following ways :—

(1) Since the angle BCA is the sum of the latitudes of the two places A and B, if $l_1$ and $l_2$ are these latitudes (north and south respectively),

$$\text{Then, } \theta + l_1 + l_2 = z_1 + z_2$$
$$\text{or, } \theta = z_1 + z_2 - l_1 - l_2.$$

(2) The angle BMA may also be obtained by direct observation.

For this, the angular distance between M and a star is observed at A; let it be $\theta_1$.

Similarly, the angular distance between M and the *same star* is observed at B; call it $\theta_2$.

Then $\theta = \theta_1 + \theta_2$, evidently, if we admit that the direction of the star (indicated by arrows in the diagram) remains unchanged, as we pass from A to B. This is true, on account of the enormous distances of stars from the observer (Ch. XII).

15. For the success of this method, the two places of observation should be as far apart as possible and (if possible) *on the same meridian.* But this may not prove convenient in practice. It is desirable, however, that the

meridians should differ as little as possible. For if there is an appreciable change of meridian, account will have to be taken of the change of declination of the moon, as it moves from one meridian to the other. But this introduces a complication, for apart from the difficulty of introducing this correction, an extremely accurate knowledge of the moon's motion is involved.

16. The observatories at Greenwich and the Cape of Good Hope satisfy the requirements of the problem extremely well, their distance in latitude being more than 85°, and their difference in longitude, less than 18°.

17. From the above, it is abundantly clear, why it is necessary for the success of this method that the stations A and B should be as far apart, as possible. For, unless this is the case, the angle AMB will not be sufficiently large to be measurable.

Even then, it is only in the case of the moon that the method is applicable, on account of its comparative nearness.

18. By measuring the parallax of the moon from day to day, from full-moon to full-moon, it is easy to verify that the distance of the moon varies, so that the path of the moon round the earth is an ellipse.

19. If we try to apply the same method for determining the distance of the sun, we should fail altogether. For the parallax of the sun is so small that such—comparatively—direct measurements will not avail. Had it not been so, the method of the Greek Astronomer, Aristarchus, would have been quite capable of giving the required result. It would, therefore, be not without interest to consider this ingenious method, although it failed—because the quantity to be measured was much too small to be observed by its means.

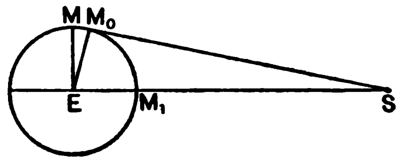



Fig. 86.

20. It is known that when the moon is just half full, (at $M_0$) (fig. 86) the angle subtended by the line joining the earth and the sun subtends a right angle at the moon. If this moment can be accurately noted, we should know one of the acute angles and, therefore, all the angles of a right-angled triangle, of which one side (*viz.*, the distance of the moon from the earth) is known, for the angle subtended by the line joining the sun and the moon at the earth is evidently proportional to the interval between new-moon ($M_1$) and the moment considered. But this interval differs so little from the period of half a lunation—only about half an hour, that it is difficult to determine it with accuracy. Aristarchus took this difference (*i.e.*, the time of describing the $\angle M_0EM$) to be twelve hours and hence got a result altogether wide of the truth.

The extreme minuteness of the angle to be determined renders it necessary that resort should be had to indirect means for its determination :

21. Our object being to observe the angle subtended at the sun by the line joining two distant places (say A and B) on the surface of the earth, we may proceed as follows :

Take another base line CD (fig. 87), which can be identified by astronomical observations, such that AC and BD meet in the sun at S ; then if we can measure the angle subtended by CD at the sun, we can determine solar parallax, provided we know the length AB.

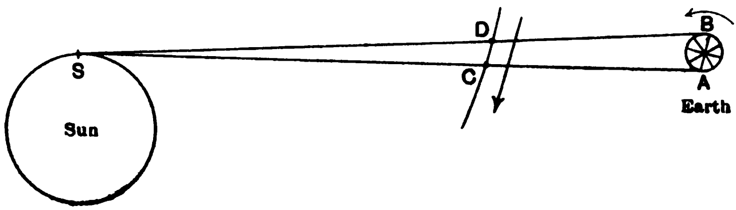



Fig. 87.

For the circular measure of this angle $= \dfrac{AB}{\text{sun's distance}}$

and parallax (in circular measure) $= \dfrac{\text{rad. of the earth}}{\text{sun's distance}}$.

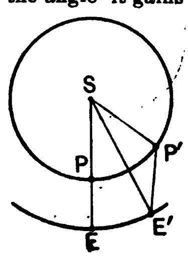
Now, the planet Venus is an opaque body which moves in an orbit, lying between the sun and the earth. It will, therefore, sometimes happen that it will appear to move across the sun's disc as a black spot. If the observers at A and B note the moments at which Venus is just entering the sun's limb, the direction in which Venus is seen by these observers will be given by AC and DB and the line DC will really represent the path of Venus, in its orbit, as *seen by an observer on the earth,* during the interval that elapses between the observations made at A and B. The rate at which Venus appears to move in its orbit (as seen by an observer on the earth) is known, for it is, in fact, the rate at which the angle subtended at the sun by the line joining the earth and Venus changes.[1] Hence, the angle

[1] Since the Synodic period of Venus is 584 days, the angle it gains on the earth in one day is $\frac{360}{584}$ degree.

This is 1·54″ per minute.

That is, if the planet moves from P to P′ (fig. 88) and the earth from E to E′ in one day, then, the angle E′SP′ $= \frac{360}{584}$ degree.

This is, then, the average rate at which Venus moves along its orbit, as seen by the earth.



subtended at the sun by CD can be calculated. Thus, the distance of the sun from the earth can be determined, provided, of course, we know the length AB. This is Delisle's method.

22. A, B are taken near the earth's equator but as far apart from each other, as possible.

In the diagram (fig. 87), the places, A, B, the equator AB, as well as the planet's orbit and S are taken in the same plane ; this is not actually the case. Allowance has, accordingly, to be made for this in the calculation. Moreover, in the above explanation, the sun's centre and the point of ingress are taken to be coincident (in order that the two diagrams (figs. 87, 88) may agree).

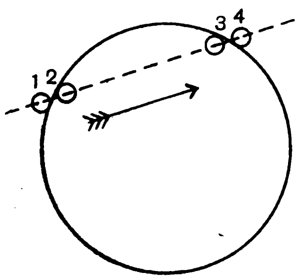



Fig. 89.
Contacts in a Transit of Venus.

23. In Halley's method, the observers at A and B note the moments of ingress (1, 2) and egress (2, 4) of the planet (fig. 89) or the whole duration of the transit.

The mode of using the results of observation is as follows :—

(1) Knowing the rate at which the elongation (fig. 88) $SE'V'$[1] changes (about $4''$ per minute[2]), we can calculate

[1] Since the path of Venus across the sun's disc (fig. 89) as seen by an observer is its *relative* path, its angular position at any moment is determined by $SE'V'$. [Fig. 88.]

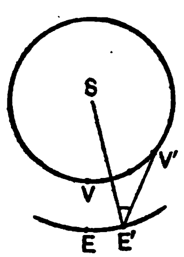



Fig. 88.

$E'$, $V'$ are corresponding positions of the earth and Venus during the transit.

[2] The rate at which Venus appears to cross the solar disc may be roughly calculated as follows :—

From fig. 88,

$$\frac{E'V'}{SV'} = \frac{\text{circular measure of } V'SE'}{\text{circular measure of } SE'V'}$$

since both the angles are small.

the angle subtended by the paths $aa'$ and $bb'$ (fig. 90) traced by Venus on the solar disc, as seen by the observers A, B.

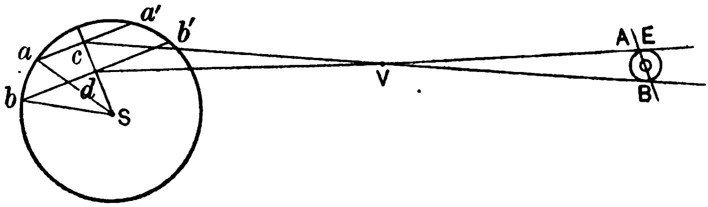



Fig. 90.—Halley's Method.

Hence, the angle subtended by $cd$ where $c$, $d$ are the middle points of $aa'$ and $bb'$ can be calculated :

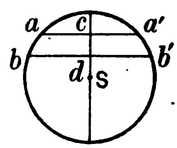



Fig. 91.

For, if R is the radius of the sun (centre S) (fig. 91)

$$\left.\begin{array}{l} R^2-\frac{1}{4}bb'^2=dS^2 \\ R^2-\frac{1}{4}aa'^2=cS^2 \end{array}\right\} \text{and } cd=dS-cS.$$

Thus, $cd$ can be calculated as an angular measure, since all other quantities in these equations except $cd$ are given in angular measure; *i.e.*, since the angles subtended at the observer's eye by the corresponding lengths are known, the angle subtended by $cd$ can be calculated.

(2) Moreover the *actual length* of $cd$ can also be calculated.

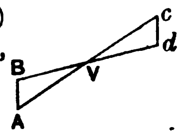
But $\frac{E'V'}{SV'}=\frac{3}{7}$ approximately. Accordingly,

$$\frac{\text{angular rate at which Venus gains on the earth}}{\text{angular rate of transit across the sun, as seen from the earth}}=\frac{3}{7}$$

Hence the angular rate of transit

$= \frac{7}{3} \times 1''\cdot55$ (art. 21 foot note) $=3''\cdot6$, approximately.

For the triangle ABV and $cd$V (fig. 90) may be regarded as similar, and, accordingly,




$$\frac{cd}{c\text{V}} = \frac{\text{AB}}{\text{AV}}$$

*i.e.*, $cd$ (in miles) = AB (in miles). $\dfrac{\text{dist. of V from the sun}}{\text{dist. of V from the earth}}$

$$= \frac{723}{277} \text{ AB (in miles) nearly.} \qquad \text{[Art. 26.]}$$

(3) Now, if any length $l$ on the sun's surface subtends at the earth, an angle, whose circular measure is $\theta$, then, we have the relation $\theta = \dfrac{l}{\text{D}}$, where D is the distance of the sun from the earth; but parallax $\text{P}_\odot = \dfrac{a}{\text{D}}$, in circular measure, $a$ being the radius of the earth.

$$\text{Hence } \frac{\theta}{\text{P}_\odot} = \frac{l}{a}.$$

Therefore, if $l'' =$ no. of seconds in $\theta$ and $\text{P}_\odot'' =$ no. of seconds in $\text{P}_\odot$,

we have $\dfrac{l''}{\text{P}_\odot''} = \dfrac{l}{a}$, where we may take $l = cd$ in miles and since its angular measure is also known, we can find $\text{P}_\odot''$.

24. Obs. When the distance of the sun from the earth has been determined, the determination of the dimensions of the solar system becomes a matter of simple calculation.

## Radii of the Sun and the Moon.

25. Let $p''$ be the equatorial horizontal parallax of the moon in seconds.

$\text{P}_\odot''$ be the parallax of the sun in seconds.

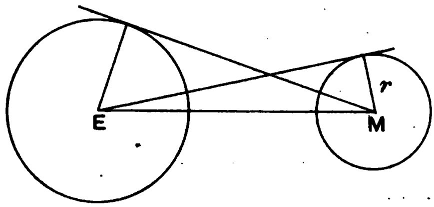
$r''$ = angular radius of the moon „ „

$\text{R}''$ = „ „ of the sun „ „

$d$ = distance of the moon, = D distance of the sun, $a$ = radius of the earth, $r$ that of the moon, R, that of sun

$\frac{a}{d} = p$ (in circular measure).

$\frac{a}{D} = P_o$ (in circular measure).




Similarly, $\frac{a}{d} \div \frac{r}{d} = \frac{p''}{r''}$. [Fig. ]

Similarly, *i.e.*, $\frac{a}{r} = \frac{p''}{r''}$ and $\frac{a}{R} = \frac{P_o''}{R''}$.

*Ex.* The sun's horizontal parallax is $8''\cdot8$, finds its distance.

Circular measure of parallax $= \frac{8\cdot8}{206265} = \frac{a}{D}$.

$$\therefore D = \frac{206265}{8\cdot5} \times 4000.$$

$= 93,700,000$ miles nearly.

if $a = 4000$ miles.

26. In the case of a planet, the following method will give its distance, if the earth's distance from the sun is known.

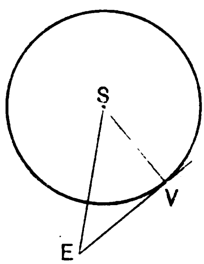



Fig. 84.

Let V (fig. 84) be an inferior planet at its greatest elongation; then, if the orbit be assumed to be circular (which will be, nearly, true for Venus but not for

Mercury) then, EV=SE cos SEV, where $\angle$ SEV is the greatest elongation. For Venus,

$\frac{EV}{SV}=\frac{277}{723}$ nearly.

Similarly, for a superior planet.

27. Another method, which is also suitable for a superior planet is as follows:

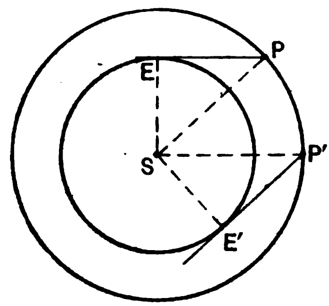



Fig. 85.

Let E′,E (fig. 85) be the positions of the earth in its orbit and P,P′, the corresponding positions of a superior planet, when *in quadrature*.

Then, $\angle$ SEP = $\angle$ SE′P′ = a right angle, if the earth's orbit is assumed to be circular.

If $t$ is the interval from E to E′, and the angles are expressed in circular measure.

$$\left.\begin{array}{l}\text{Then } \angle ESE' = w_e t \\ \text{and } \angle PSP' = w_p t\end{array}\right\}$$

$\therefore \angle ESE' - \angle PSS' = (w_e - w_p)\, t;$

also $2\pi = (w_e - w_p)\, S$,

if $w_e$ = angular velocity of the earth,

$w_p$ = „ „ of the planet,

and S = synodic period.

Again $\angle ESE' - \angle PSP' = 2\angle ESP$, if the orbit of P is assumed to be circular.

$\therefore\ 2\angle ESP = \frac{2\pi t}{S}$ is known, since $t$ can be determined as well as S.

But tan ESP $=\frac{EP}{ES}$: whence, EP can be determined *as well as SP*, if ES is known, by one of the methods, already described. The ratio ES : SP may also be assumed known, from Kepler's Third law.

## CHAPTER XII

### ANNUAL PARALLAX

1. Having measured the distances of the members of the solar system, we proceed next to measure the distances of stars. These distances are so great, that any base line on the surface of the earth is quite inadequate for the purpose. In fact, even the distance of the earth from the sun is wholly inadequate for any but the nearest of them. Taking actual figures, the maximum angle subtended by the radius of the earth's orbit at a star being defined as *secular* or **annual parallax** of the star (fig. 94), it is found that *Aldeberran*

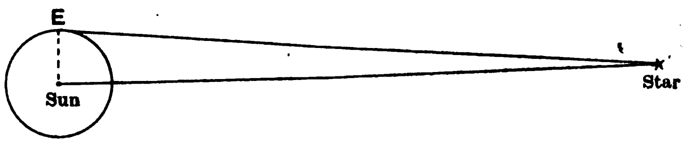



Fig. 94.

has only a parallax of ·116 second and *Canis Minor* ·226 second. In other words, the distance of the former is 1,890,000 times and that of the latter, 819,000 times the distance of the sun, which, itself, is more than 93 million miles. It follows from this, that if we assume *Aldeberran* to be even as large as the sun, its angular radius will be only a millionth part of that of the sun, so that, unless the magnifying power of our telescopes is enormously increased, a star must still appear to be a mere point, even if seen through the most powerful telescope.[1]

[1] It is interesting to note that Neptune, the outermost member of the solar system is at a distance of only 30 times that of the sun. The distance of the nearest stars is, thus, great, in comparison with the furthest member of the solar system. It is, hence, easy to conclude that there is an enormous void, separating the solar system from its nearest neighbour.

2. Let us now consider how secular parallax is taken account of, in correcting stellar positions.

Let E (fig. 95) be the position of the Earth in its orbit and P′, that of a star ; then EP′ is the direction in which it is seen *from* E. As E changes its position, this direction changes, while the direction in which it is seen from the sun remains unchanged, as the positions of both the sun and the star are fixed. Completing the parallelogram, PP′SE, we observe that EP is a fixed direction in space.

With E as centre, describe the celestial sphere of the

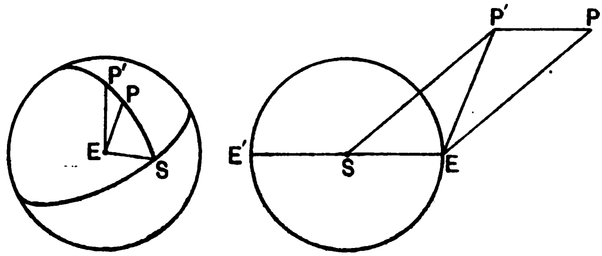



Fig. 95.

observer and draw EP′, EP and ES parallel to these lines in the second figure. Then, EP is the fixed direction of the star, EP′, the direction, as seen by an observer, carried with the earth in its orbit and, accordingly, PP′ is the displacement of the star on account of annual parallax. Again, since E, S, P, P′, are in one plane *through* E, in the second figure, P, P′, S are on a great circle, in the first. Accordingly, this displacement is easily seen to be towards S, the position of the sun on the celestial sphere.

Let the angular displacement PEP′ be equal to $p$.

Then, from the triangle P′EP (second fig. 95),

$$\frac{\sin p}{SE} = \frac{\sin PP'E}{PE}.$$

$$\therefore p = \sin \text{PP}'\text{E}.\frac{\text{SE}}{\text{PE}}$$

$$= \sin \text{PES}. \frac{\text{distance of the sun from the earth}}{\text{distance of the star from the sun}},$$

since the $\angle$s PP′E and PES differ by the infinitely small angle $p$.

The maximum value of $p$ is called the **Annual parallax** of the star.

Let this be $p_0$.

Then, $p_0 = \dfrac{\text{distance of the sun from the earth}}{\text{distance of the star from the sun}}$ and is the **Annual Parallax.**

3. To determine the annual parallax of a star, we may proceed by a method, similar to that employed to determine the diurnal parallax of the moon.

[We have already considered a method of determining the annual parallax of a superior planet in art. 27, ch. X, where we determined the angle SPE (fig 85), when $\angle$SEP is 90° and this is the annual parallax of the planet.]

For this, a faint star, very near the one whose annual parallax is to be determined, is selected and the angular distances between the two are measured, when the earth is at E and E′, two diametrically opposite points of the earth's orbit. [In other words, the diameter of the earth's orbit is taken as the *base line.*]

Let these be $\theta_1$ and $\theta_2$.

Then, if we can assume that the faint star is so remote that it has no parallax, i.e., *the lines of sight to it from E and E′ are parallel,* then evidently,

$$\theta_1 + \theta_2 = \text{EP}'\text{E}' = 2p \text{ very nearly.}$$

4. From the mode of representation, on the celestial sphere, of the displacement due to parallax, it obviously follows that the effect is similar to that due to aberration.



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And we may at once deduce that

(1) the path of the displaced position of a star due to parallax, taken throughout the year is an ellipse, whose major axis is $p_0$ (or the annual parallax) and minor axis is $p_0 \sin \beta$ (where $\beta$ = celestial latitude);

(2) that the path reduces to a circle, when the star is at the pole of the ecliptic, and

(3) that it is a straight line, when it is in the plane of the ecliptic.

5. Let us now compare the different causes, which produce an apparent change in the position of a body.

These are

(1) refraction,

(2) diurnal parallax,

(3) annual parallax,

(4) aberration,

(5) precession and nutation.

The effect of all these is to produce an apparent displacement of the body:

(1) Refraction displaces it along the vertical through the star, *away from the zenith.*

(2) Diurnal parallax also produces displacement along the vertical, but *towards the zenith.*

(3) Annual parallax displaces the body along the great circle through the star and the sun, towards the point, which the sun occupies at the time.

(4) Aberration displaces it towards the point 90° behind the sun, along the great circle through the star and this point.

(5) Finally, precession produces a change in the longitude of all celestial bodies, by the same amount, while nutation produces a small periodic change in both latitude and longitude.

When all these changes have been allowed for, it is still found that the observed displacements are not wholly accounted for. This must, accordingly, be due to the **proper motion** of these stars. In other words, the so-called *fixed* stars are not really fixed but have *motions of their own.*

### EXERCISE.

1. Taking the moon's horizontal parallax to be 57′ and its angular diameter 32′, show that the moon's radius is 1,123 miles nearly.

2. The synodic period of mercury being given, find the angle gained per minute by mercury on the earth round the sun (as centre), on an average.

3. The sun's horizontal parallax being taken to be 8″·8 and his angular diameter, equal to 32′, find its diameter in miles.

4. Find the angular velocity with which mercury crosses the sun's disc, assuming the ratio of the distances of mercury and the earth to be as given by Bode's law.

5. Why is it not strictly true that the azimuth of a heavenly body is unaffected by parallax?

6. What is the distance of a star, of which the parallax is 2″?

7. The annual parallax of a double star is 0″·307, and the apparent angular distance between its two members is 15″. Find the distance of the double star and the distance between its two members.

8. Show that if $p_0$ is annual parallax in seconds of arc, the distance of a star is $206265 \dfrac{R}{p_0}$, where R is the distance of the earth from the sun.

9. Show that if D is equal to the distance of a star in light-years, then $D = \dfrac{3.262}{p_0''}$. where a light-year = distance that light travels in a year (velocity of light 186000 miles, per second).

## CHAPTER XII.

### Eclipses

1. From the phases of the moon, it is easy to conclude that the moon is an opaque body. That being so, as the earth goes round the sun, and the moon, round the earth, it will, sometimes, happen that the moon, coming between the sun and the earth, will cut off the sun's light—partially or wholly. We shall, then, have a **solar eclipse** (fig. 96). Similarly, if the moon enters the shadow cast by the earth, it will be partially or wholly invisible from the earth. There will, then, be a **lunar eclipse**. [Fig. 97.]

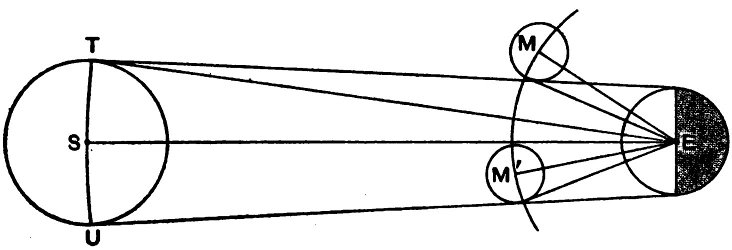



**Fig. 96, representing a solar eclipse.**

2. We propose to consider the conditions and circumstances of a lunar and a solar eclipse.

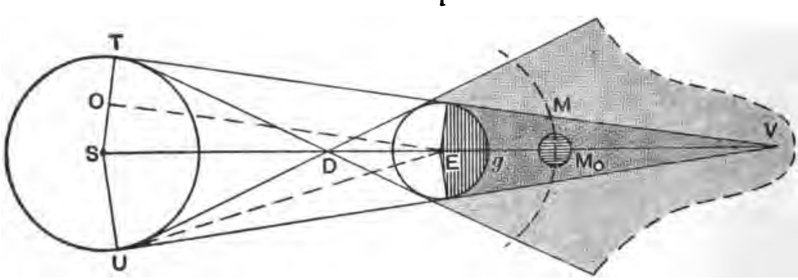



**Fig. 97.**
**Lunar eclipse.**

3. It will be evident, at once, that a solar eclipse can only take place at a new moon, for it is only then, that the moon comes between the sun and the earth, while a lunar eclipse can only occur at a full moon.

4. Again, at a lunar eclipse, the moon actually enters the shadow of the earth, so that, *to all places* on the earth's surface, at which, the moon would be otherwise visible (*i. e.*, at all places at which the moon is actually above the horizon), the moon will be eclipsed, whereas in the case of a solar eclipse, the nature of the phenomena will depend on the position of the observer.

5. Now, if the orbits of the earth and the moon were coplanar, there would be an eclipse of the sun, at *every* new moon and an eclipse of the moon, at every full moon. But as these orbits are in different planes, an eclipse can only take place, if the new moon or the full moon occurs either *at* a node or at a point, sufficiently near to a node. The reason of this will, presently, appear.

6. As the earth moves round the sun, a cone of shadow moves in space. If we take a normal section (art. 8, note) of this cone, *at the distance of the moon*, we may imagine this section to move, as a disc of shadow (cast by the earth), with its centre in the plane of the earth's orbit. At the same time, the lunar disc, as presented to the observer (full at opposition) is moving with its centre along the lunar orbit. We may, therefore, replace the motions of the moon and the cone of shadow by those of the lunar disc and the disc of shadow. Now, as the orbits of the moon and the earth are inclined to each other, it may happen that these discs do not overlap each other at all, at a particular full moon, either partially or wholly. There will, in that case, be no eclipse. If, however, they do overlap, there will be an eclipse—a partial eclipse, if they

overlap partially and a total eclipse if the shadow overlaps the lunar disc, completely. We proceed to consider these various cases.

7. Let N (fig. 98) be the node of the lunar orbit and NE, a portion of the path traced out by the centre of the earth's shadow at the distance of the moon (which must evidently be similar to the ecliptic).

Let NM be a portion of the lunar orbit, E, the centre of the shadow, M, the centre of the moon, when these are *in opposition in longtitude* (*i. e.*, when they have the same longitude), in the neighbourhood of a node.

It will be presently seen (art. 14) that NE and NM are elementary lengths, so that N, E, M may be taken to be coplanar; also, $\angle$ NEM $=90°$, since (E, M, having the same longitude), EM must be perp. to NE.

8. As both the discs[1] are in motion, superpose a motion in longitude, $n'n$ equal and opposite to the hourly change in longitude of the earth's shadow on both [$n'm$, $m$M being the hourly changes in longitude and latitude of the moon].

Then, E may be held to be at rest and M to move along its relative path MN′. If we now draw EL perp. from E on this relative path, EL will be the shortest distance between the centres of the two discs during the actual motion.

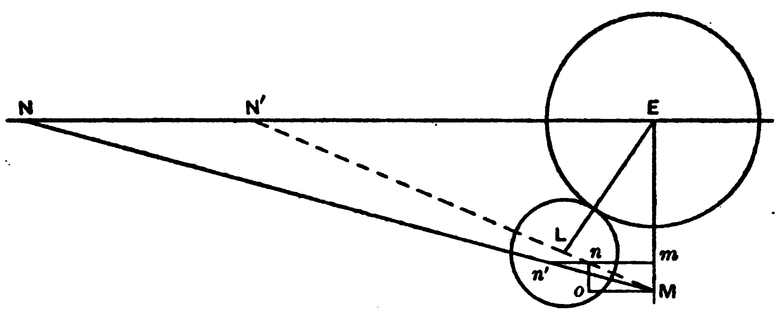



Fig. 98.

[1] These discs are really sections of the cone of shadow and the moon by the plane NEM.

9. (1) If, therefore, EL (drawn perp. from E on N′M) (fig. 99), is greater than the sum of the radii of the shadow and the lunar disc, there cannot be an eclipse.

(2) If it is less than this sum, there must be an eclipse.

(3) If this distance is less than the difference of the radii, there will be a total eclipse. [Fig. 101.]

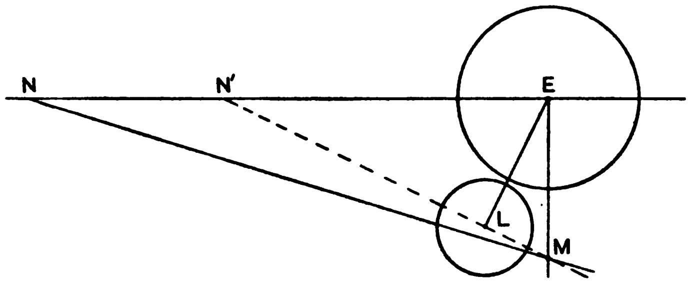



Fig. 99.

10. Hence, if E, as centre and radius, equal to the sum of the radii of the two discs, we describe a circle, the points ($M_1$, $M_2$) (fig. 100) at which it intersects MN′ (if it does at all[1]) are the points of first and last contact of the moon with the shadow, at a partial eclipse.

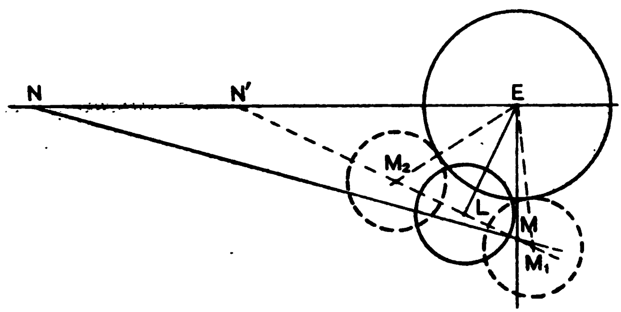



Fig. 100. Progress of a partial eclipse.

If with E, as centre and radius, equal to the difference of the radii, a circle is described, the points ($M_3$

[1] Otherwise, there will be no eclipse.

$M_4$) (fig. 101) at which it will intersect N′M (if it does)[1] will be the position of the centre of the lunar disc for the beginning and end of the *total* eclipse.

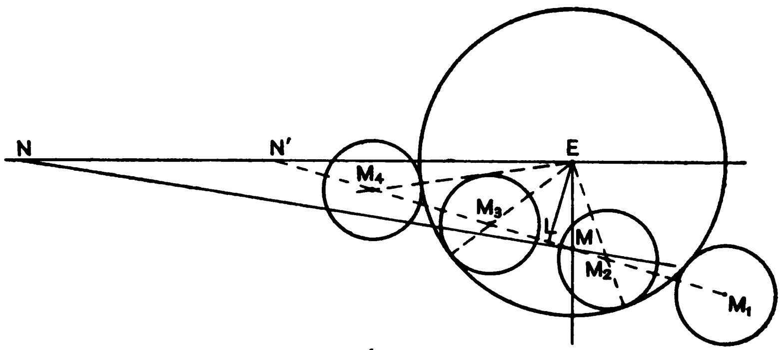



Fig. 101. Progress of a total eclipse.

$M_1$, $M_4$ (where $EM_1 = EM_4$ = the sum of the radii on N′M) mark the positions of the centre of the lunar disc for the beginning and end of the eclipse.

In every case, L marks the middle of the eclipse.

11. The condition for an eclipse and its nature are thus dependent on

(1) the length N′E which depends on the interval between moon's passage through the node and the full moon;

(2) the size of the shadow (at the distance of the moon) and that of the lunar disc, as presented to the observer.

12. To find the size of the shadow, at the distance of the moon at a full moon, we proceed as follows:

Let S be the centre of the sun and E (fig. 97) that of the earth, $\overset{\frown}{TV}$, one of the common direct tangents to the sun and the earth, V, the apex of the cone formed by the direct tangents, and EM, the distance of the moon from the earth.

[1] Otherwise, the eclipse cannot be total.

Then, the angular radius of the shadow

$$
\begin{aligned}
&= \angle \mathrm{MEV} \\
&= \angle \mathrm{EMT} - \angle \mathrm{MVE} \\
&= p_m - (\angle \mathrm{TES} - \angle \mathrm{VTE}) \\
&= p_m - \odot + p_s
\end{aligned}
$$

where $p_m$ = the horizontal parallax of the moon,

$\odot$ = semi-diameter of the sun,

$p_s$ = the parallax of the sun, all expressed in circular measure.

13. It is found, however, that in calculating the condition for an eclipse, this quantity has to be increased by $\frac{1}{60}$th of its value, in order that the calculated results should agree with those observed. This is, evidently, due to the fact that the solar rays, passing through the lower strata of the earth's atmosphere are quenched by absorption, so that the cone of shadow practically touches a larger sphere than the earth.

14. The size of the shadow, as just reduced changes in value, with the change in the distances of the sun and the moon from the earth. The angular radius of the lunar disc (as seen by an observer on the surface of the earth) changes also, having in fact, a maximum and a minimum value. We have, moreover, seen that a lunar eclipse cannot take place, unless EL (fig. 99) is less than the sum of the radii of the shadow and the moon.

*Ecliptic limits.*—The value of NE or N′E (for they are very nearly equal) corresponding to this limiting value of EL gives the **ecliptic limit**. There will be a **major** ecliptic limit, so that if NE is greater than this value, an eclipse cannot take place and a minor ecliptic limit, such that, if NE is less than this quantity, an eclipse *must* take place. It is, moreover, easy to see that NE is of the same order of quantities as the size

of the shadow and the lunar disc. In other words, NE is an elementary length [Art. 7.]

15. For a solar eclipse, the lunar disc itself forms the disc of shadow, while the section of the cone touching the earth and the sun, at the distance of the moon is the illuminated disc. When this disc is shut out of view by the interposition of the shadow or the lunar disc, there is a solar eclipse—partial or total.

16. To find the size of this illuminated disc, we notice that its angular radius is

$$=p_m+\odot-p_s. \quad \text{[Fig. 96.]}$$

17. In the case of a solar eclipse, however, the position of the observer makes a considerable difference in the character of the eclipse observed and the conditions required.

18. Thus, let S be the sun, M the moon and

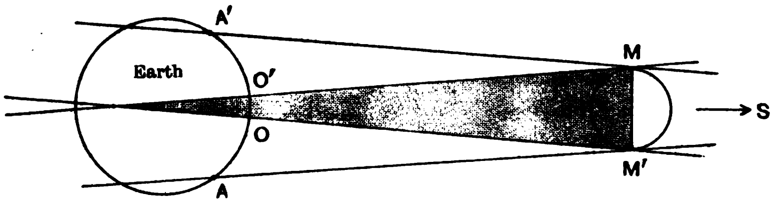



Fig. 102. Total eclipse of the sun.

AOA', the earth (fig. 102). Let also OO' be the region of the earth, common (at any time) to the earth and the direct tangent cone[1] enveloping both the moon and the sun, and AA' the region, common to the earth and the *conjugate* cone; then, as both the moon and the earth are in motion, these will trace out belts on the earth and it is easily seen (by drawing a tangent cone to the moon from the observer) that for any place within the belt OO', the eclipse will be total, that for any place

[1] If lines are drawn touching two spheres, so as to meet at a point, away from the centres of both, they form the direct tangent cone. If they meet *between* the centres, they from the conjugate cone.

beyond OO′ but within AA′, the eclipse will be partial, while for places beyond AA′, there will be no eclipse.

19. Again, if the direct tangent cone to the sun and the moon meets the earth along the belt PQ (fig. 103), then for any place within this belt, only the central portion of the sun is eclipsed. The eclipse is, in this case, called **annular**.

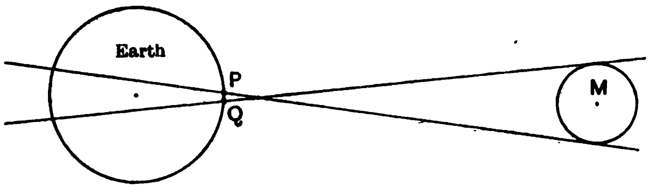



Fig. 103.

20. We have seen that an eclipse will take place, if the relative positions of the sun, the moon, the earth and the moon's nodes have certain defined configurations.[1]

Now the moon's nodes have a retrograde motion, completing relatively to the observer, a cycle in 346·62 days. Thus, in a period of about 6585 days[2] or 18 years

[1] In other words when the moon is in opposition at or near a node, as seen by the earth.

[2] The synodic period of the revolution of the moon's nodes is 346·62 days, *i.e.*, relatively to an observer on the surface of the earth, a revolution of the nodes is completed in 346·62 days. Also, one lunation occupies 29·53059 days. That is, the moon is in opposition to the sun, as seen by an observer on the earth's surface, in each succeeding period of 29·53059 days.

The same relative positions of the sun, the moon, the earth and the lunar node (suitable for an eclipse) will therefore, recur in a period which is as nearly as possible the least common multiple of these two quantities. This period is about 6585 days.

For 223 lunations

$= 223 \times 29{\cdot}53$

$= 6585$

$= 19 \times 346{\cdot}62$

$= 19$ synodic revolutions of the node.

and 11 days (or 10 days, if there are five leap years in this period), the node, as well as the sun, the moon and the earth will come back to the same relative positions as at the beginning.

It will follow, therefore, that if a series of eclipses take place, during an epoch of 223 lunation or 6585 days, eclipses will recur in the same order and practically under similar circumstances, during each succeeding period of 6585 days.

21. This was the *saros* (lit. repetition) of the Chaldeans. By means of a table, carefully prepared, of all the eclipses during any one such period, it was possible to predict all future eclipses, with a fair degree of approximation as to dates and circumstances.

22. *Frequency of eclipses.* In order to determine the frequency of eclipses during a year, we have to bear in mind that the solar ecliptic limits are about $15° 32'$ and $18° 36'$, while the lunar ecliptic limits are $9\frac{1}{2}°$ and $12°$.

Again, in the interval between a new moon and a full moon, *i.e.*, $14\frac{3}{4}$ days, the sun moves through $15\frac{1}{3}°$ $(= 14\frac{3}{4} \times 62'19'')$ on an average, relatively to the node, which is less than the minor ecliptic limit for a solar eclipse but greater than the major ecliptic limit for a lunar eclipse.

Hence, there *must* be a solar eclipse in the neighbourhood of a node. It can be shown, however, that there may be as many as three eclipses near a node—one lunar and two solar (if a *full moon* occurs exactly at a node, for instance).

Thus, at each passage of the sun, through the node, there may be as many as 3 eclipses, 2 of the sun and 1 of the moon. And there must be, at least, one solar eclipse.

We observe, further, that while the sun's passage from one node to the next is completed in 173 days, six lunations

comprise a period of about 177 days. It follows, accordingly, that the number of eclipses are somewhat unevenly distributed and it can be shown[1] that there cannot be more than 7 eclipses in the year.

23. We have seen that when the moon enters the cone, touching the sun and the earth, a considerable portion —and under certain circumstances—the whole of the sun is eclipsed. This is, obviously, due to the comparative proximity of the moon to the earth. In the case of a more distant body like Venus and Mercury, the phenomenon will be different. They appear to cross the solar disc as a black spot.

[1] Assuming a daily advance of the sun, relative to the node of 62′, let us suppose, in the first place, the full moon to occur exactly at a node. Then, at the previous and following new moon, the angular distance of the sun from the node, on an average will be 15° 20′. As this is less than the major ecliptic limit for a solar eclipse, there may be two solar eclipses at that node and three eclipses altogether, including the lunar eclipse at the node itself.

At the next node, the full moon will occur 4 days after passage through the node, at an angular distance from the node which is within the minor ecliptic limit for a lunar eclipse. There will thus be a lunar eclipse at the full moon. Moreover, since the previous new moon will occur at an angular distance of 11° 12′, there will be a solar eclipse also at this new moon, but at the following new moon, occurring, as it does at an angular distance of 19° 29′, from the node, there will be no eclipse.

At the first node again, on the completion of the cycle, the full moon will occur, 8 days after passage through the node, *i.e.*, at the angular distance of 8° 16′ of the shadow from the node. There will, therefore, be a lunar eclipse at this node also, preceded by a solar eclipse at the previous new moon but not followed by another.

Thus, in 354 days (from full moon at a node to full moon 8 days after passage through the same node), there may be six eclipses.

If we count the eclipse at the new moon preceding the first full-moon, we get seven eclipses in 368½ days.

If, however, the full moon occurs, two days before the sun's passage through the node, then the previous new moon will occur, at an angular distance of 13° 15′ from the node, while the following new moon will

The general conditions for this to happen are of course the same as in the case of a solar eclipse:

They must be in inferior conjunction at or near enough to a node.

On account of the small size of these planets, as seen by the earth, as well as the smaller obliquity of their orbits, the conjunction required for a transit must occur much nearer to the node than in the case of the solar eclipse. This leads to an interesting result.

Taking the case of Venus, since the period of its revolution round the sun is 224·7 days, we have

since 8 years

$$= 8 \times 365{\cdot}256 = 2922 \text{ days nearly}$$
$$= 13 \times 224{\cdot}7 + 1 \text{ day (nearly)}$$

two transits may occur at an interval of 8 years.

Thus, if a transit occurs exactly at a node in any year, there may be one at the same node, 8 years after, as the next conjunction occurs sufficiently near to the node (within 1 day) but the next conjunction (16 years after) will occur 1·8 days later than its passage through the node. This will make it impossible for the transit to occur.

We have further

$$235 \text{ years} = 235 \times 365{\cdot}265$$
$$= 85835 \text{ days}$$
$$= 382 \times 224{\cdot}7 \text{ very nearly,}$$

so that in every 235 years, a transit must occur at the same node.

occur at an angular distance of 17°24′, making three eclipses (2 solar, 1 lunar) possible. At the next node, there will also be three, while at the original node, after completion of the cycle, there will be two.

These eight eclipses will cover 368½ days but there will be seven in 354 days. Taking account of all possible cases, in this way, it is found that the maximum number of eclipses possible in a year is seven.

We conclude, accordingly, that there cannot be more than seven eclipses in a year, nor less than two.

This being the next number after 8, for which the above relation holds, we conclude that, at the same node, a transit must be followed or preceded by a transit at an interval of 8 years but that the next transit cannot take place till 235 years, after.

'The following table gives the actual occurrences of these transits:

| Ascending Node. | Diff. | Descending node. | Diff. |
|---|---|---|---|
| Dec. 6,1631 A.D. | | | |
| | 8 years | | |
| Dec. 4,1,639 | | June 1761 | |
| | 275 years | | 8 years |
| Dec. 1874 | | June 1769 | |
| | 8 years | | 235 years |
| Dec. 1882 | | 2004 | |

transils of mercury are more frequent than those of Venus, since its periodic time is only 87·96 days nearly.

### EXERCISE

1. In an eclipse, does the obscuration begin on the eastern or the western limb of the body eclipsed? In a solar eclipse, does the shadow of the moon move eastward or westward on the earth's surface?

2. In a total lunar eclipse, given the size of the shadow at opposition in longitude, the size of the lunar disc, as well as the latitude of the moon at opposition, draw a diagram to determine the progress of the eclipse, the rate of motion of both the moon and the earth, being given.

3. Show by means of a diagram the effect of a change in the moon's motion on the duration of an eclipse.

## CHAPTER XIII

### TIME

1. In order to measure a physical quantity, we have to choose a suitable unit, in terms of which the quantity has to be measured and devise means, whereby the given quantity and the unit chosen may be compared, so as to find out how many times, this quantity contains the unit.

2. The unit chosen must satisfy the following criteria :

(1) It must be of the same kind, as the quantity to be measured.

(2) It must be an invariable quantity.

(3) It must be easily procurable or accessible, or must be capable of being easily identified.

3. The unit of time used in Astronomy for scientific purposes is the **sidereal day** ; that is, the period of the earth's rotation about its axis, and is therefore, equal to the interval between the successive transits of a star across the meridian of a place.

4. It is obvious, in the first place, that the 3rd criterion specified above is satisfied by the unit.

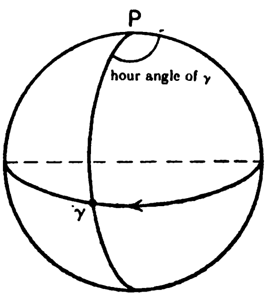
5. Before, however, we can be satisfied that it is really suitable for our purpose, we must consider (1) how any given interval of time may be compared with it and (2) what grounds there are for supposing that it is a constant interval of time.

Both these purposes are served by the **Astronomical clock.**

This is a clock, which, as we have seen (art. 14, ch. iv) keeps sidereal time. In other words, the interval of time, indicated by it is a sidereal interval, so that its hour-hand sweeps out 360° in a sidereal day of 24 sidereal hours.

6. Let us assume, now, that the clock is correct; that is, that the period of oscillation of the pendulum or of the spring is constant and equal to two sidereal seconds. Then, a simple observation is enough to show that the interval between the successive passages of the same star and those of all stars are the same. This proves that a sidereal day is a constant interval of time. Moreover, an interval measured by such a clock gives the magnitude of any sidereal interval, with any desired degree of accuracy.

7. *Def.* The Local sidereal time at any place is the sidereal interval that has elapsed, since the preceding transit of the first point of Aries, across the meridian of the place. It is, therefore, equal to the hour angle of the first point of Aries (as an inspection of fig. shows).




8. In order to determine the local sidereal time, therefore, it will be necessary, in the first place to *set* the astronomical clock, so that it may indicate $0^h, 0^m, 0^s$, when the first point of Aries is on the meridian. If there were a star exactly occupying the first point of Aries, the setting of the clock would have been a matter of comparative ease. As, however, there is no such star, we have to resort to indirect methods. [Ch. VIII.]

9. Let us recall that the procedure followed is to select a star whose R.A. is known and observe it at its meridian passage and at the same time set the clock, so

that the time it indicates at the moment is equal to the R.A. of the star.

This, however, requires that the R.A. of a star should be known, independently of the clock and we have seen how this is done. When the clock has been set, the time indicated by it at any moment, is the local sidereal time, if the clock is correct. Otherwise, a correction has to be applied, called the error of the clock.

10. Although a sidereal day is a suitable unit of time for astronomical purposes, it is not a convenient unit for practical purposes. For the first point of Aries is on the meridian, *i.e.*, it is $0^h\ 0^m\ 0^s$ by the sidereal clock, at midday on the 21st March, at sun-rise on 22nd June, at midnight on 23rd September (when the sun's R.A. is 180°) and in the evening, *i.e.*, at sun-set on 22nd December.

11. The practical inconvenience of this mode of reckoning time is, thus, apparent. Since, in fact, it is by the motion of the sun that our daily lives are ordered, it is easy to see that a practically useful unit of time should have reference to this motion.

The **solar day**, that is, the interval between the successive transits of the sun, across the meridian of a place, would, accordingly, appear to be the natural unit of time, but for the fact that it is not a constant interval.

We conclude, accordingly, that the unit of time that would be practically useful must satisfy the following conditions:

(1) It must, necessarily, be a constant interval.

(2) The unit must never differ except by a few minutes from a solar day.

(3) The number of these units in the year should be equal to the number of solar days in the year.

Hence, the unit should clearly be the *mean* of all the solar days in the year.

12. Such a unit is called the **mean solar day.** A clock that keeps mean solar time, *i.e.*, 24 hours (mean solar) of a mean solar day is called a **mean solar clock,** It is, in fact simply, our *ordinary* clock.

Thus, the time at any moment by such a clock would be, very nearly, equal to the *hour angle of the sun* but not exactly. It is equal to the hour angle of an imaginary body called the Mean Sun. [Art. 17.]

13. The time that has elapsed since the preceding meridian passage of the sun is called the **apparent time.**

The difference between solar time and apparent time is a small quantity. Still it requires careful investigation, in order that the time by the ordinary clock and the apparent time may be compared.

This difference is called the **equation** of **time,** so that **Clock time − apparent time = equation** of **time.**

We shall see that the apparent time can be determined by means of a sun-dial. Hence, equation of time can also be defined as *the difference between clock-time and dial-time.*

14. In actual practice, while the unit of time is the mean solar day, another unit (called the **Civil Day)** is taken, which is equal to it but *is reckoned from midnight to midnight,* so that our ordinary clocks point to twelve, when the mean sun is in the meridian.

But as the dial face is divided into 12 parts, mean time, as given by the clock is still proportional to the hour angle of the mean sun, up to midnight, even as a mean solar clock ought to do, if we followed the mode of reckoning, as with the Astronomical clock.

15. To find the relation between the sidereal day and the solar day.

We know that the sun is at $\gamma$ on the 21st of March. Hence, the sun and $\gamma$ will be at the meridian together on that day.

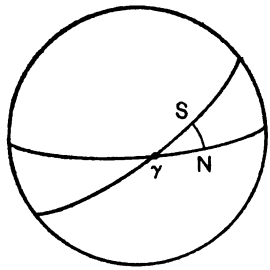



Fig. 103.

On account of the diurnal rotation of the earth, $\gamma$ will be again at the meridian of the place at the end of one sidereal day but in that interval, the sun will have moved on, from $\gamma$, so that the meridian will have to rotate through 360° *plus* the change of R.A. of the sun in one solar day, before it can over-take the sun. Hence,

one solar day = one sidereal day + the sidereal interval corresponding to the sun's change of R.A. in one solar day. (1)

It follows, therefore, that if the number of solar days in the year is equal to N, then,

N solar days = N sidereal days + the sidereal interval corresponding to the sun's change of R.A. in the year (*i.e.*, 360° or 24 hours).

But as this last is equal to one sidereal day, we have

$$N \text{ solar days} = (N+1) \text{ sidereal days.} \qquad (2)$$

That is, the number of sidereal days in the year is one more than the number of solar days, in the year.

But the number of sidereal days in a year can be obtained by observation. For this, we have only to find by the astronomical clock, the time taken by the sun to complete its circuit round the earth.

This, of course, can be obtained with any degree of accuracy.

Thus, we get the value of $N+1$ and, hence, the number of solar days in the year is known.

16. It follows also from equation (1) (art. 15) that—

(1) since the sun's change of R.A. is not uniform, the solar day is not a constant interval of time, and

(2) if the motion of the sun were such that its change of R.A., while completing its revolution in a year had been uniform, then, the solar day would have been a constant interval of time and would have been the mean of all the (actual) solar days in the year.

17. Imagine, then, a body moving uniformly along the equator, with the mean angular velocity of the true or actual sun. Then, the interval between the successive passages of such a body across the meridian of a place will be equal to the mean solar day.

This imaginary body is called the **mean sun**. Hence, the moment, at which the mean sun transits across the meridian (*i.e.*, $12^h$, by the ordinary clock) is the **local mean noon** and *per contra,* the time of the true sun's meridian passage is called **apparent noon.**

18. It follows, accordingly,

(1) the mean time is the hour angle of the mean sun, expressed in time at the rate of 15° to one mean solar hour,

(2) the number of mean solar days in the year is equal to one more than the number of sidereal days.

*Hence,*

$$\frac{\text{One mean solar day}}{\text{One sidereal day}}=\frac{366{\cdot}242216}{365{\cdot}242216}.$$

19. We have seen that the equation of time is a small quantity, being the difference between clock-time and apparent time. If our clocks could always be relied on to give correct time, this quantity—at any rate, corresponding to *apparent noon* could be found by simply noting the clock-time of the true sun's meridian passage. The difference between this and 12 hours would then give the equation of time (positive, if this time is less than 12). Similarly the difference between clock-time and the hour-angle of the true sun would give the equation of time at any moment.

20. For this, it is necessary to set the clock correctly, Moreover, no clock can always keep correct time ; we have accordingly to determine the equation of time, independently of the clock (by calculation) and thus *set* and *correct* the clock. We proceed now to consider how the equation of time arises and varies throughout the year. The actual calculations are given in advanced treatises.

21. It will be observed that the difference between the solar and mean solar day arises from the fact that while the sun moves along the ecliptic with non-uniform motion, the mean sun moves along the equator, uniformly. The difference thus arises from two causes :

(1) On account of the fact that the motion of the sun is along the ecliptic, instead of being along the equator, *i.e.*, on account of *the obliquity of the ecliptic* ;

(2) that the motion of the sun (along the ecliptic) is variable, *i.e.*, " on account of *unequal motion.*"

22. The effect due to each of these being small, we may consider each separately and obtain the resultant effect by adding them together algebraically.

In dealing with them, therefore, we may regard only one alone operative and the other, for the time being, non-existent.

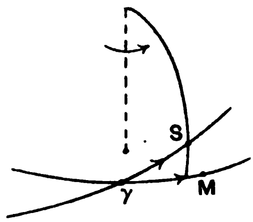



Fig. 104.

23. THE EFFECT OF OBLIQUITY.—Since, in considering the effect of obliquity, alone, we are to neglect the effect of unequal motion, and since the mean sun moves uniformly along the equator, by definition, another imaginary body must be supposed to move, *uniformly along the ecliptic*, at the same rate.

Let us suppose that they start together at $\gamma$ and let the diagram represent the celestial sphere of an observer supposed to be at the centre of the earth.

Then if we take $\gamma S = \gamma M$, then, the mean sun will be at M, when the true sun is at S (since both are moving uniformly at the same rate).

Hence, the declination circle through S is always nearer $\gamma$ than that through M, while the sun moves from vernal equinox to summer solstice (for Northern latitudes).

Now, the diurnal motion of the earth is in the same direction as the apparent annual motion of the sun. Hence, any meridian, as it is carried round on account of the earth's diurnal motion will first have the true sun and then the mean sun transiting across it.[1] That is, the apparent

[1] Since the celestial sphere represents the celestial vault, we may imagine $\gamma M, \gamma S$ to be two fixed curves, traced on the celestial vault. The earth will then be a concentric sphere, rotating about the normal to the plane of $\gamma M$.

or true noon will *precede* mean noon. Therefore, mean time will be less than the apparent time. Or, the equation of time, is *negative*. The same is true from autumnal equinox to winter solstice, while from (either) solstice to (either) equinox, it is positive.

At $\gamma$ and $\Omega$ as well as at the solstices, this part of the equation of time is evidently zero, and calculation shows that its maximum values are $= \pm$ 10 minutes, nearly.

24. (2) The effect of UNEQUAL MOTION.

We must now imagine the "mean sun"[1] to move along the ecliptic with uniform angular velocity and the true sun to move along the ecliptic with unequal[2] angular velocity, which follows Kepler's 2nd law.

As the starting point of the mean sun has not so far been defined, let the two suns start together at perigee, where this portion of the equation of time will then vanish.

Then, since, at perigee, the true sun's (apparent) angular velocity is maximum, the true sun will go ahead of the mean sun and, accordingly, any meridian will have first the mean sun and then the true sun, over it; that is, the mean noon will precede true noon or, in other words, mean time will be greater than the apparent time. The equation of time will, accordingly, be positive from perigee to apogee. But the equation of time is again zero at apogee and, accordingly, there must be a moment between these two epochs, at which this portion of the equation of time is maximum. Calculation shows that the maximum value occurs about the end of March and its value is, then, 7·7 minutes nearly. Similarly, from apogee to perigee, the equation is negative.

[1] Or the imaginary body which was requisitioned in the previous discussion.

[2] And opposite to that of the earth in its orbit.

25. On the whole, we have the following results :—

The equation of time due to obliquity is zero, on 21st March, 22nd June, 23rd September, 22nd December.

and max= 10m in February and August

and= −10m in May and November.

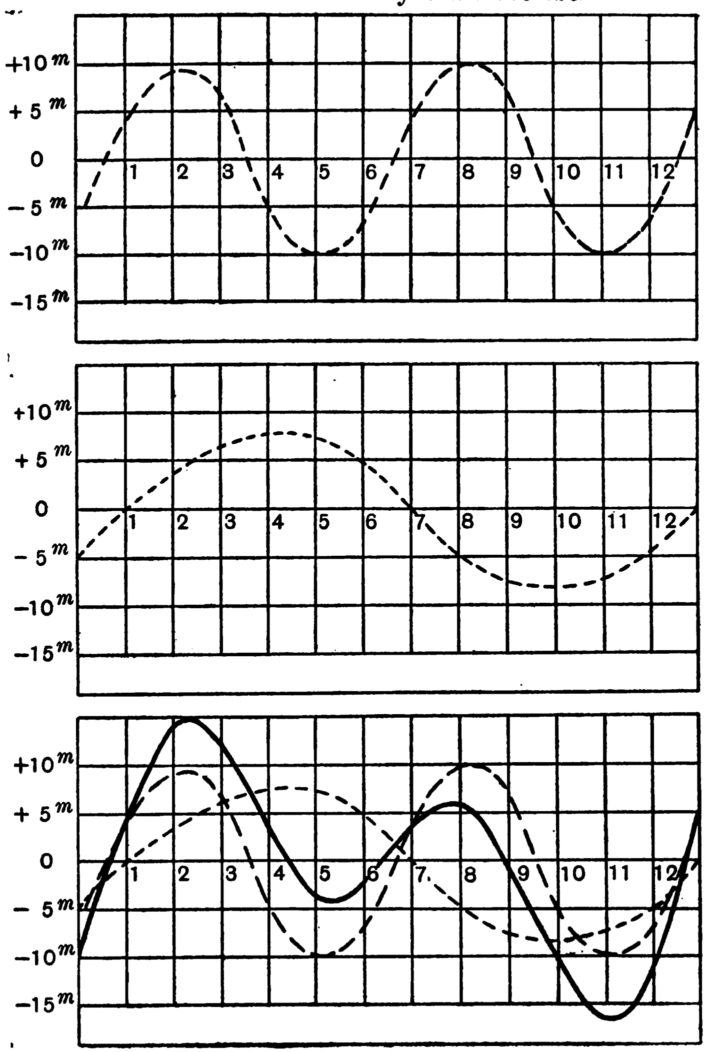



**Fig. 105 (1), 105 (2), 105 (3).—The Equation of Time.**

The actual results are plotted on curve, fig. 105 (1).
The equation of time due to

unequal motion $=0$ on 31st December, and 1st July.
and max. $=7 \cdot 7$, about the end of March
and $= -7 \cdot 7$ at the end of September.

Plotting these on curve (2) and adding the ordinates of curves (1) and (2), we find that the equation of time vanishes four times, a year, *viz.*,—On or about April 15, June 15, September 1, and December 24th.

*Time by Observation.*

26. Time is determined by observation as follows :—

Observe the clock time of the sun's meridian passage.

Then the difference between the clock time and 12 should be equal to the equation of time for the local apparent noon, at the place of observation. If this is not the case, the clock is in error and the correction required is known.

The Nautical Almanac gives the equation of time corresponding to Greenwich mean noon as well as Greenwich apparent noon for each day and its hourly variation. Hence, in order to obtain the equation required, in the preceding paragraph, it is necessary to know the Greenwich time corresponding to the local time considered. This is easily obtained.

For each 15° of west Longitude makes a difference of 1 hour in time. Thus, if M is the local mean or apparent time and L is the west Longitude of the place.

Then $M + \frac{L}{15} =$ the corresponding Greenwich time (mean or apparent).

27. To find the local sidereal time, it is only necessary to observe the transit of a known star; then the R.A. of the star is equal to the local sidereal time at the moment of observation.

28. *Conversion of time.*

1. To convert a given sidereal interval into the corresponding mean solar interval and *vice versa.*

For this, it is only necessary to remember that 365·242216 mean solar days are equivalent to 366·242216 sidereal days.

Thus, one sidereal day $= \frac{365 \cdot 242216}{366 \cdot 242216}$ solar day. Hence, if $M_h$ = number of mean solar hours in a given interval of time, and $S_h$ = number of sidereal hours in the same interval, then, $\frac{M_h}{365 \cdot 242216} = \frac{S_h}{366 \cdot 242216}$

$\therefore M_h = S_h(1-n)$ when $n = \frac{1}{366 \cdot 242216} = \cdot 00273043$;

also $S_h = M_h(1+n')$, where $n' = \frac{1}{365 \cdot 242216}$ ; $= \cdot 00273791$

and $(1+n')\ (1-n) = 1$.

Again, one sidereal day contains
$23^h\ 56^m\ 4^s \cdot 0906 = 24^h - 3^m\ 55^s \cdot 9094$ (mean).

One mean solar day contains
$24^h\ 3^m\ 56^s 5554$ (sidereal) $= 24^h + 3^m\ 56^s 5554$

Thus, the factor $n$ produces a change of $3^m\ 55^s \cdot 9094$ in 24 hrs. or $9^s \cdot 8296$ per hour.

Similarly, the factor $n'$ produces a change of $3^m$ $56^s \cdot 555$ in 24 hrs. or $9^s \cdot 8565$ per hour.

Ex. 1. Express $16^h$ (mean) as a sideral interval. Ans. $162^m$ $37^s \cdot 704$.

2. To convert mean solar time at any place or local mean time into local sidereal time and *vice versâ.*

If $\gamma$ is the first point of aries, then the hour angle of $\gamma$ = local sidereal time.

Also the hour angle of the mean sun M is the local mean time. But $\gamma$M = R.A. of M. Then, from the figure 106, where PAB is the celestial sphere of the place of observation, it is evident that

sidereal time = mean time + mean sun's R.A.

For, if P is the celestial pole and $\gamma$M, the equator and PAB, the celestial meridian,

$$\begin{aligned} \text{then } \gamma PA &= \text{hour angle of } \gamma \\ &= \text{local sidereal time} \\ \text{and } MA &= \text{hour angle of the mean sun} \\ &= \text{local mean time} \end{aligned}$$

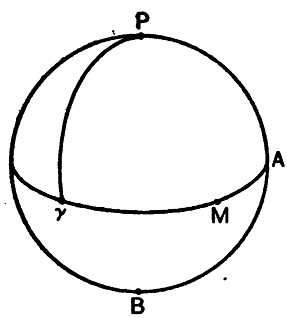



Fig. 106.

or if S = sidereal time,
M = mean time,
R = mean sun's R.A.

Then, $S = M + R$.

But R.A. of the mean sun is given in the Nautical Almanac for Greenwich mean moon from day to day. Also, since R.A. of the mean sun changes uniformly, throughout the year,

change of R.A. of the mean sun in 1 day

$$= \frac{24}{365 \cdot 2412} = 24n'$$

*i.e.*, change of R.A. of the mean sun in 1 hour $= n'$

Now if L = west longitude of the place of observation, $M + \frac{L}{15}$ = mean time at Greenwich, corresponding to the meantime (M) of observation at the place.

$\therefore$ if $R_o$ = R.A. of mean sun at Greenwich at *previous mean* noon,

$$\text{then, } R = R_o + \left(M + \frac{L}{15}\right) n,'$$

$$\text{whence, } S = M + R_o + \left(M + \frac{L}{15}\right) n' \qquad (1)$$

$$\text{and } M = \frac{S - R_o - n' \frac{L}{15}}{1 + n'} = S - R_o - n\left(S - R_o + \frac{L}{15}\right) \quad (2)$$

$$\text{since } \frac{1}{1-n} = 1 + n' \text{ and therefore } \frac{n'}{1+n'} = n$$

The first equation gives the sidereal time, when the mean time is given, the second, the mean time, the sidereal time being given.

Ex 1. On 1909, Feb. 18th, the sidereal time at Greenwich mean moon is $21^h\ 51.^m\ 13^s{\cdot}55$. Show that the transit of the first point of Aries takes place at $2^h\ 8^m\ 25^s{\cdot}35$ mean time.

We have, $S = M(1 + n') + R_o$, since $L = 0$

Here $R_o = 21^h\ 51^m\ 13^s\ {\cdot}55$

$\therefore$ when $S = 0$, *i.e.*, when the first point of Aries transits across the meridian,

$$(1 + n')M + R_o = 0 \text{ or } = 24$$

$$\begin{aligned}\therefore \quad M &= (24 - 21^h\ 51^m\ 13^s{\cdot}55)(1 - n)\\ &= (2^h\ 8^m\ 46^s{\cdot}45)\ (1 - n)\\ &= 2^h\ 8^m\ 46^s{\cdot}45 - 21{\cdot}095\\ &= 2^h\ 8^m\ 25^s{\cdot}35 \quad \text{nearly}\end{aligned}$$

**Ex. 2. Columbia College, New York, is in longitude $4^h\ 55^m\ 54^s$, west of Greenwich. The sidereal time of mean noon at Greenwich on 1903, Dec. 12 is $17^h\ 23^m\ 8^s$. Show that on the same day, when the sidereal time at Columbia College is $20^h\ 8^m\ 4^s$, the local mean time is $2^h\ 43^m\ 41^s$**

$$\text{We have } M = S - R_0 - n\left((S - R_0 + \frac{L}{15}\right),$$

$$S = 20^h\ 8^m\ 4^s$$

$$\text{and } R_0 = 17^h\ 23^m\ 8^s \text{ and } \frac{L}{15} = 4^h\ 55^m\ 54^s.$$

3. To convert apparent solar time into mean solar time.

This is obtained from the equation

**Clock time − dial time = equation of time,** provided we know the equation of time.

Now, the equation of time for each day is given in N.A for Greenwich mean noon, as well as apparent noon. For any other time, the necessary correction is also given—as difference, which gives the rate at which the equation of time changes, per hour. Hence, the equation of time for Greenwich time (mean or apparent) can be calculated. Finally, since any given local time can be converted into corresponding Greenwich time, the equation for any local time can be deduced.

4. The relation between local time and Greenwich time.

This is at once deduce[1] from the fact that each 15° of longitude corresponds to a difference of 1 hour so that—

Local time $+\frac{L}{15}$ = Greenwich time,[1] where L is the west longitude of the plane.

Ex. The longitude of Calcutta is $88^\circ\ 20'\ 1''\cdot 79$ E.; find the local time corresponding to Greenwich noon.

[1] Is true for sidereal time as well as solar time, for it depends on the diurnal rotation of the earth, relative to a star or the sun.

29. *Length of morning and afternoon.* One effect of the equation of time is noteworthy.

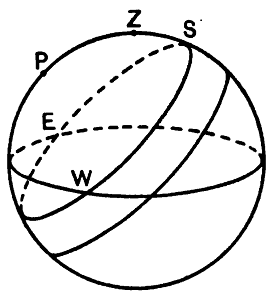


Fig. 107.

Assuming the diurnal circle of the sun to be a parallel of the celestial equator ESW (*i.e.*, neglecting a small change in the declination of the sun during one day), it easily follows from the diagram (fig. 107), that the interval from E to S is equal to that from S to W, where E and W mark the positions at sunrise and sunset and S marks the position, at the meridian passage, of the sun's centre.

That is, the interval from sun-rise to the apparent noon is equal to the interval from apparent noon to sunset. Let this interval be $=\mathrm{I}$.

If then, the morning is reckoned from sunrise to mean noon, this interval is equal $\mathrm{I}+\mathrm{E}$ where E is the equation of the time.

Similarly, the afternoon is equal to $\mathrm{I}-\mathrm{E}$.

Hence, morning—afternoon $=2$ E.

The actual length of the morning and the afternoon depends on the declination of the sun and therefore on the time of the year.

30. All these modes of reckoning refer to the meridian of the observer. When we wish to obviate this difficulty, we may reckon time from a fixed epoch, say, vernal equinox. Time, so measured is called **equinoctial time**; that is, it is the interval (measured in mean solar units) that has elapsed since preceding vernal equinox.

31. *Def.* The time taken by the earth to complete its revolution round the sun is called the **year**.

According to the points of reference, however, we have to distinguish different kinds of years. Thus, we may reckon the cycle completed when the Earth goes from a fixed point in space, a star, for instance back to the same star. The period is, then, called a **sidereal year.**

Or, we may take the period from one vernal equinox to the next, *i.e.* (on the geocentric view), from the moment when the sun is at $\gamma$ to the moment, when it comes to $\gamma$ again. This period is called the **tropical** year. If $\gamma$ were a fixed point in space, the tropical year would be equal to the sidereal year. But this is not the case (on account of precession).

Finally, we may reckon this period from the moment at which the earth is in perihelion to the moment when it comes back to it. This is called the **anomalistic** year.

If the major axis of the earth's orbit had been a fixed direction in space, this would be the same as the sidereal year. As this is not the case, it also differs from the sidereal year.

On account of the retrograde motion of the first point Aries at the annual rate of $50''\cdot22$ and progressive motion of the apse-line at the rate of $11''\cdot25$ per year, the relation between the different kinds of years is given by the following equations, *viz.*:—

$$\frac{360^\circ - 50\cdot''22}{\text{tropical year}} = \frac{360^\circ}{\text{sidereal year}} = \frac{360^\circ + 11''\cdot25}{\text{anomalistic year}}\cdot$$

32. We have already seen how the tropical year can be determined by observation. It is, in fact, equal to 365·242216 mean solar days. Hence, the sidereal year is equal to 365·256374 mean solar days and the anomalistic year to 365·259544 mean solar days.

33. **The Calendar.** Just as the sun's apparent daily motion regulates our daily lives, the tropical year, which

marks the recurrence of the seasons would seem to be the natural long-period unit of time, as it is with reference to the recurrence of the seasons that our lives are regulated. But for practical convenience, such a long-period unit of time should consist of an exact number of days.[1]

34. Accordingly, the long-period unit of time is chosen so that (1) it should consist of an exact number of days, and (2) the beginning and end of this period shall never differ much from the beginning and end of the tropical year.

This is called the **civil year.**

35. These two conditions are satisfied, as far as possible by adopting the following mode of reckoning:

(1) An ordinary year is taken to consist of 365 days, while a *leap year* to consist of 366 days.

(2) Every 4th year is a leap year, so that a year which is divisible by 4 is a leap year, except the centuries that are not divisible by 400, these being taken to be ordinary years.

36. These conventions are based on the following calculation:—

A tropical year consists of $365^d\ 5^h\ 48^m\ 45^s{\cdot}5$.

Hence 4 tropical years $=365^d \times 4 + 23^h\ 15^m 02^s$.

Hence, three ordinary years and one leap year (in every 4 tropical years) differ from four tropical years by $44^m 53^s$ only and accordingly, the introduction of the leap year (due to Julius Cæsar) produces a nearly complete adjustment. This is called the **Julian Calendar.**

But the above difference, small as it is, accumulates in course of centuries, so that in 400 years, it becomes nearly equal to 3 days.

[1] A little consideration will show, how almost in every case, a difficulty will arise, if this did not happen to be the case; *e.g.*, in calculating age, yearly profit, yearly salary, etc.

Hence, in order that 400 years of the (corrected) calendar should be equal (nearly) to 400 tropical years, it is necessary to drop 3 leap years in the course of every 400 years. This can be most, simply, effected by regarding only those century years which are divisible by 400 as the leap years, the other centuries being only reckoned as ordinary years. This reformed calendar was due to Pope Gregory and is called the **Gregorian Calendar.**

37. This small residual difference will accumulate into about one day in 2,000 years and may therefore, for the present, be neglected.

38. *Sun dial.*

Apparent time, or time indicated by the motion of the true sun is evidently given by the hour angle of the true sun. Now, as the sun moves on the celestial vault, the shadow cast by a style on any chosen plane will move with it, so that the shadow, the style and the sun are always in the same plane. In other words, the shadow is the inter-section of the chosen plane with the plane containing the style and the sun. Hence, the angular position of the shadow will depend on the hour angle of the sun and will enable us to measure this hour angle, at any time. This will enable us to determine the apparent time.

If the chosen plane coincides with that of the celestial equator, at the place of observation and a style is placed in the direction of the polar axis and hence perpendicular to the chosen plane, then, it is clear that the angle that the shadow makes with its position at midday is equal to the hour angle of the sun and since, on our definition, apparent time is simply proportional to this hour angle, we get the apparent time by simply reading off this angle.

As the sun's motion is not uniform, the hour angle is not strictly proportional to apparent time but we have agreed to define it so, the error committed being allowed for all practical purposes (for which we only require mean time) by the correction which is called the equation of time.

If the chosen plane is other than that of the celestial equator, we have to take the projection of the shadow on this plane and the corresponding angles made by the shadow have to be determined by calculation.

### EXERCISE.

1. The longitude of Dublin being $6^\circ\ 40'$ W., find the time in Dublin, when it is 2 P.M. at Greenwich.

2. At Madras in longitude $80^\circ\ 14'\ 19''\cdot5$ East, an observation is made on September, 6th, 1865 at $9^h\ 21^m\ 12^s\ 8$ mean time; find the corresponding sidereal time.

If the sidereal time is $20^h\ 24^m\ 13^s\cdot72$, find the corresponding meantime.

3. Find the R.A. of the true sun at true noon on the 30th of January, being given the following; equation of time at mean moon on the 30th of January and sidereal time of mean noon on the same date.

4. At New York in longitude $74^\circ\ 1'\frac{1}{4}$ W., an observation is made at $7^h\ 15^m\ 10^s$, mean solar time on a certain day on which the sidereal time of mean noon at Greenwich is $10^h\ 15^m\ 54^s$; find the corresponding sidereal time.

5. The times of sun-rise and sun-set on November 1st are found from the tables to be $6^h\ 56^m$ and $4^h\ 32^m$ respectively. Find approximately the equation of time.

6. Assuming that the maximum amount of the equation of time due to obliquity exceeds the maximum of that due to eccentricity, show that the equation vanishes four times in the year.

7. The mean time being 4 hours, find the corresponding sidereal time, given the sun's mean daily motion to be $59^m\ 8^s33$ and the R.A. of the preceding mean noon $144^\circ$.

8. If in a certain system of calendar, the leap year recurs every third year, find how the adjustment may be effected, so that in $n$ years, it may be nearly complete.

## CHAPTER XV

### THE POSITION OF A PLACE ON THE EARTH'S SURFACE

*Latitude by Observation*

1. Observe a circumpolar star at its upper and lower transit across the meridian. Then, if $a_1$, $a_2$ be the altitudes of the star at these transits, then

$$\text{Latitude} = \frac{a_1 + a_2}{2}.$$

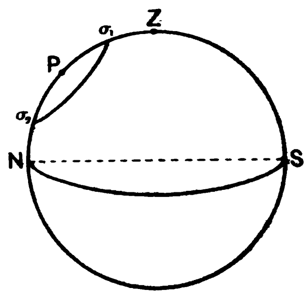



Fig. 108.

For $N\sigma_1 = a_1 \quad N\sigma_2 = a_2$ if $\sigma_1, \sigma_2$ are the positions of the star at upper and lower culmination, fig. 108, N being the north point.

Then, $NP + P\sigma_1 = a_1$

$NP - P\sigma_2 = a_2$;

$\therefore \ 2NP = a_1 + a_2 \quad$ since $P\sigma_1 = P\sigma_2$.

This method is only suitable for use in a fixed observatory. It, however, does not require a knowledge of the declination of the star.

2. If the declination of the star is known, one observation will suffice.

For then $P\sigma_1 = P\sigma_2 =$ declination ($\delta$).

Hence, $NP = \text{lat.} = a_1 - \delta$.

The same method will apply, if any other body, the sun for instance, is observed (provided, of course, its declination is known).

Obs. In the case of the sun, the meridian passage of the sun's centre has to be noted.

3. At sea, the only useful method is that of observation of the meridian altitude of the sun or a star—in preference, the former. As the meridian altitude is the

maximum altitude on any given date, observations are taken at short intervals, commencing before apparent noon till the maximum is passed. As the change in altitude is then very slow, the effect of a change of position, on account of the motion of the ship has comparatively little effect. It is, moreover, of no importance, as an absolutely correct determination of latitude is not required.

When the sun is the body observed, its declination has to be calculated from the Nautical Almanac. This gives the sun's declination for each day of Greenwich mean noon and its hourly variation. The chronometers carried by the ship give Greenwich time and hence the interval between Greenwich mean noon and the local apparent noon is known. Thus, the declination of the sun corresponding to local noon (*i.e.*, time of maximum altitude) can be calculated.

| | | |
|---|---|---|
| Ex. The sun's declination on previous Greenwich noon | = | 22°. |
| Chronometer time | = | $6^h$. |
| Horary change | = | 19″·5. |
| Hence, the sun's declination at local noon | = | 22° 1′ 57″. |

### *Longitude by Observation.*

4. Longitude referred to any prime-meridian, Greenwich for instance, is known, if we know the Greenwich mean time and the corresponding local mean time. We must, therefore, have some means of knowing Greenwich mean time, at the moment at which local mean time is also determined.

(*a*) By the chronometer. If we have a chronometer which gives Greenwich time, then if the local time is 6 P.M. and the Greenwich time is, say, 2 P.M., the longitude = 60° E.

(*b*) By means of a suitably chosen celestial phenomena. If the chronometer fails, we may note the local mean time

at the moment when any celestial phenomena occurs and if the mean time of the same phenomenon is given in the nautical almanac, since this time is always Greenwich mean time, we have a means of determining the longitude.

Two such phenomena are of especial importance:

(1) Lunar distances.

(2) Occultation of a star by the moon.

The nautical almanac gives a series of tables giving the moon's distance from certain bright stars and planets (as seen by an observer at the centre of the earth) for every third hour of Greenwich mean time.

If, then, the distance of the moon, from a given star is noted and corrected for parallax, we may find by interpolation or by a direct reference to the tables, the Greenwich mean time at the moment of observation. And, thus, the longitude is determined.

Similar methods will apply to the case of an occultation. But these are not suitable for use at sea.

(*c*) By electric telegraph. If the Greenwich mean time is signalled by electric telegraph to a station at which the local mean time is also noted at the same time, the longitude can be determined. At sea, the signal is transmitted by wireless.

5. It will be observed that the determination of longitude depends on a determination of the local time.

Obs. If the local apparent time is determined, the local mean time can be calculated as in Art. 26 (3), Ch. XIII.

(1) The readiest method is to determine the local sidereal time, by the meridian transit of a known star.

But this can be only availed of, in a fixed observatory.

(2) Since, generally, the time corresponding to meridian altitude, that is the maximum altitude of the

sun is apparent noon, this is the simplest method for use at sea.

6. When both the latitude and longitude have been determined, the position of the place is known.

Such a method, however, is hardly suitable for use at sea. The following simple method, due to Capt. Sumner is the one in common use.

For this, it is necessary to remember that at each moment, the sun is vertically overhead at some point on the surface of the earth called the *subsolar point.* Moreover, the angular distance of this place from any other place is equal to the zenith distance of the sun, as observed at the latter, at the moment considered.

This being premised, let P be the subsolar point, at any moment. With P, as centre and radius equal to the arc of a great circle which subtends at the centre of the terrestrial globe, an angle equal to the observed zenith distance of the sun, describe a small circle on the globe. Then, the place of observation must be on this small circle.

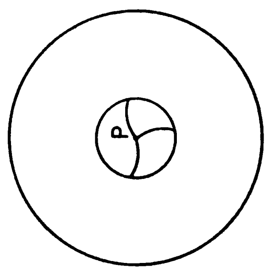



Fig. 109.

Let Q (not shown on the diagram) be the subsolar point, a few hours later; then, if we describe a second great circle having for its radius, the zenith distance of the sun at the second observation, the intersections of the two circles are the only two possible positions of the place of observation, provided the observer (or the ship) is at rest. It would then be easy to tell which of the two points represents the actual position of the ship, either from a knowledge of its approximate position or from a rough observation of the azimuth of the sun.

If the ship is in motion, let AB, represent on the scale of the globe the run of the ship during the interval between two observations (in direction and magnitude). Then it will be necessary to lay off a length AB in direction and magnitude, such that A is on the first circle and B on the second. Then A must be the first position of the observer, and B, the second.

As before, we get two such pairs of positions and we have to select one pair, on similar considerations as the above.

# ANSWERS.

## CHAPTER I.

2. 3492 miles.

3. 45°.

## CHAPTER III.

4. 213° 21′; $33^h\ 44^m\ 52^s$.

5. 11°·6.

6. $\omega h \sqrt{\frac{2h}{g}}$; when $l=90°$.

7. 698·4 miles.

12. At the same time.

13. 0°. R.A.=0. H.A.=90°, Z.D=90°; the latter two will vary.

## CHAPTER IV.

1. Half the diff. of the readings gives the meridian altitude. Latitude being known $\delta$ is given by

$$\text{colat} \pm \delta = \text{altitude}.$$

2. $l=65°\ 18'\ 10''$
   $\delta=77°\ 25'\ 25''$

Observation is to be made by Alt-azimuth.

3. 59° 27′ 50″.

4. $12^h\ 34^m$.

5. $\begin{cases} l=56°\ 2'\ 30''. \\ \delta=53°\ 47'\ 30''. \end{cases}$

6. $\delta_1=20°$.
   $\delta_2=10°$.

## CHAPTER V.

1. 6 A.M. 6 P.M.
3. At the Summer solstice 83° 28′.
   „ „ Winter „ 46° 32′.
   „ „ Equinoxes 60°.
4. 38° 7′.
5. $\begin{cases} 55° \; 1'. \\ 7° \;\; 5'. \end{cases}$
6. 10°·8, 90.
14. 72000 years.

## CHAPTER VI.

5. (*i*) 52 min. nearly, (*ii*) 50 min.
9. 49·9 ft.
11. 32·68 days.
13. ·06.
14. 2156 miles.

## CHAPTER VII.

1. Inferior: half the radius of the earth's orbit.
2. (*a*) Superior, (*b*) superior, (*c*) superior or inferior, (*d*) inferior, (*e*) inferior, (*f*) inferior.
3. ·93.
4. ·75.
5. 4325 days.
6. 579 days.
8. 1·6 times the velocity of the earth.
9. (*i*) $1 : 2^{\frac{1}{3}}$; (*ii*) 15040 miles.
10. 6 miles.

## CHAPTER VIII.

1. R.A. of the star $= 18 \cdot 5$ hours
and that of the sun $= 3 \cdot 5$ and $8 \cdot 5$ hours respectively.

4. $27^\circ\ 1'\ 4'' \cdot 64$ in 1921 A.D.

5. The plane containing the axis of the spinning-top and the vertical line about which it revolves.

## CHAPTER X.

1. The latitude $l$ is given by

$$180^\circ - 2l = 55^\circ + 58'' \cdot 2\ (\tan 25^\circ + \tan 30^\circ).$$

2. $\text{Sin}^{-1} \dfrac{5}{13}\ - 2'\ 19'' \cdot 7.$

3. Apply formulæ

$$180^\circ - 2l = 30^\circ + 40^\circ + k\ (\tan 30^\circ + \tan 40^\circ).$$
$$180^\circ - 2l = 25^\circ + 45^\circ + k\ (\tan 25^\circ + \tan 45^\circ).$$

4. $2l = 30^\circ + 40^\circ + k\ (\tan 30^\circ + \tan 40^\circ)$
where $k =$ coeff. of refraction.

## CHAPTER XI.

3. $\dfrac{360^\circ}{T}$ where T is the No. of minutes in the Synodic period.

4. 872000 miles.
$0^c \cdot 024$ per hour.

7. 11972190 million miles.

8. $77994723 \cdot 1$ million miles.
2215 million miles.

## CHAPTER XIII.

1. $1^h\ 33^m\ 40^s$ P.M.

2. For R.A. of the mean sun, consult Nautical almanac.

Sid. time $20^h\ 24^m\ 13^s{\cdot}72$.

Mean time $9^h\ 21^m\ 12^s{\cdot}8$.

4. $3^h\ 2^m\ 42^s{\cdot}4$.

5. $-16$ minutes.

7. $4^h + 9^h\ 36^m + {\cdot}00273791\ (4^\circ + l)$

where $l$ = longitude west of the place.

# INDEX

N.B.—*The numbers refer to the pages.*